A Wake-Up Call Theory of Contagion*

Toni Ahnert† Christoph Bertsch‡

Revised November 2021

Abstract

We offer a theory of financial contagion based on the information choice of investors after observing a financial crisis elsewhere. We study global coordination games of regime change in two regions linked by an initially unobserved macro shock. A crisis in region 1 is a wake-up call to investors in region 2. It induces them to reassess the regional fundamental and acquire information about the macro shock. Contagion can occur even after investors learn that region 2 has no ex-post exposure to region 1. We explore normative and testable implications of the model. In particular, our results rationalize evidence about contagious currency crises and bank runs after wake-up calls and provide some guidance for future empirical work.

Keywords: wake-up call, information choice, financial crises, contagion, bank run, global games, fundamental re-assessment.

JEL Classification: D83, F3, G01, G21.

*We wish to thank Christine Parlour (the editor), two anonymous referees, our discussants Ana Babus, Denis Gorea, Harald Hau, Julian Kolm, Kristoffer Nimark, Richard Rosen, Laura Veldkamp, and Sergio Vicente. We are gateful for comments from Viral Acharya, Jason Allen, Kartik Anand, Elena Carletti, Amil Dasgupta, Michael Ehrmann, Douglas Gale, Piero Gottardi, Antonio Guarino, Christian Hellwig, Frank Heinemann, Ralf Meisenzahl, Alessandro Pavan, Morten Ravn, Wolfram Richter, Jean-Charles Rochet, Nikita Roketskiy, Lars Svensson, Michal Szkup, Dimitri Vayanos, Xin Zhang and seminar participants at FIRS 2017, ASSA 2015, Bank of Canada, Barcelona GSE Summer Forum 2014 (Information and Market Frictions; Experimental and Theoretical Macroeconomics), Behavorial Aspects in Macro and Finance conference 2014, Federal Reserve Board of Governors, CEA 2014, DIW Berlin, TU Dortmund, EEA 2013, EFA 2014, EFA Day-Ahead 2015, EUI Florence, Fed St. Louis, IMF, IWH Halle, London FIT at LBS, MPI Bonn, NBER Summer Institute 2014 in International Finance and Macroeconomics, NFA 2014, NORMAC 2013, RES 2014, Sveriges Riksbank, University College London, UB Barcelona, and Uppsala University. All remaining errors are our own. Earlier versions were circulated as "A wake-up call: information contagion and speculative currency attacks" and "A wake-up call: information contagion and strategic uncertainty". First version: June 2012. The views expressed in this paper are ours and should not be interpreted as reflecting the official views of any institution that we are affiliated with. We acknowledge outstanding research assistance by Jessica Redeschnig and Omar Abdelrahman.

†Corresponding author: European Central Bank, Directorate General Research, Financial Research Division, Sonnemannstrasse 20, 60314 Frankfurt, Germany and Bank of Canada and Centre for Economic Policy Research. E-mail: toni.ahnert@ecb.int.

‡Sveriges Riksbank, Research Division, Brunkebergstorg 11, SE-103 37 Stockholm, Sweden. E-mail: mail@christophbertsch.com.

1 Introduction

Understanding the causes of financial contagion is an important question in banking and international finance. A popular explanation is wake-up call contagion (Forbes 2012). According to the wake-up call hypothesis informally described by Goldstein (1998), a financial crisis in one region is a wake-up call to investors that induces them to re-assess and inquire about the fundamentals of other regions. In particular, investors acquire information about exposures to potential vulnerabilities shared with the crisis region. Such a re-appraisal of risk can lead to a contagious spread of a financial crisis across regions. Despite its popularity, including among policymakers, there has been little theoretical work on the wake-up call hypothesis. This paper aims to close this gap. We propose a theory of wake-up call contagion and derive several testable and normative implications.

A financial crisis is modelled as a standard global coordination game of regime change (Carlsson and van Damme 1993; Morris and Shin 2003; Vives 2005). A financial crisis comprises a currency attack, a bank run, or a debt crisis and occurs when enough investors act against a regime by attacking a currency peg, withdrawing funds from a bank, or refusing to roll over debt. In contrast to the standard setup, our model has two regions. Regional fundamentals are linked via the exposure to a macro shock, which captures common vulnerabilities and institutional similarities of regions. These include, for currency attacks and sovereign debt crises, the sustainability of specific institutional and developmental models as well as structural and industrial policies in emerging economies (Dasgupta et al., 2011) that can make a country prone to adverse changes in the (international) macroeconomic environment (Corsetti et al. 1999). For bank runs, these include potential interbank exposures, institutional similarities in corporate governance and proneness to financial misconduct (e.g., involvement in Libor scandal), the quality of regulation, and the credibility of deposit insurance and government guarantees that can give rise to financial sector vulnerabilities.

The potential link between regional fundamentals is commonly known ex-ante but its actual relevance is only learned upon a fundamental re-assessment. That is, we differentiate between ex-ante and ex-post exposure. The timing of the game is sequential in order to capture the notion of contagion. Investors in region 1 decide whether to act against the regime, which determines the outcome in region 1. Next, investors in region 2 observe whether a regime change occurred in region 1, update their beliefs about the common macro shock, and decide whether to learn at a cost about the ex-post exposure of region 2's fundamentals to region 1 via the macro shock. Finally, investors in region 2 decide whether to act against the regime. This setup allows us to study how information acquisition shapes investor re-assessment of fundamentals and ultimately contagion.

We find that when crises are rare and the macro shock is negatively skewed, investors in region 2 have a higher incentive to acquire information after the wake-up call of observing a crisis in region 1 (Proposition 2). Intuitively, the negative skewness creates an asymmetry that makes it more valuable to acquire information after the rare event of observing a crisis. For an intermediate range of information costs, investors learn about the macro shock if and only if a crisis occurred in region 1 (Proposition 1). This differential information choice is at the core of the fundamental re-assessment and shapes contagion. It builds on a Bayesian updating result where observing a crisis in region 1 can whip around probabilities of tail events and focus investor attention on rarely observed downside risk. Specifically, with a negatively skewed macro shock investors face an elevated risk of a strongly negative macro shock after a wake-up call, while the negative shock is less likely after no crisis. Upon a wake-up call, an investor's benefit of tailoring its attack rule to the realized macro shock is highest, so the value of learning is higher after a crisis in region 1.

We define contagion as a higher probability of a crisis in region 2 after a crisis in region 1, relative to no crisis in region 1. We isolate the wake-up call component of contagion and show that contagion can occur even if all investors learn that the macro shock is zero (Proposition 3). That is, the probability of a crisis in region 2 after a crisis in region 1 and learning that region 2 has no ex-post exposure to region 1 is higher than the probability of a crisis in region 2 after no crisis in region 1. This result consists of two parts: information choice of investors and contagion. Endogenous information is critical for the contagion result that, in turn, is driven by Bayesian updating about the shock. Observing a crisis in region 1 and learning about a zero macro shock means that region 2 has no ex-post exposure to region 1 via the macro shock. In contrast, absent a crisis in region 1, investors in region 2 choose not to acquire information and form a more optimistic belief about the shock. Hence, the probability of a crisis in region 2 is lower after no crisis in region 1.

The wake-up call contagion effect described above is positively associated with the extent of ex-ante exposure across regions. Intuitively, the degree to which investors in region 2 are more optimistic about the macro shock absent a crisis in region 1 is higher for higher ex-ante exposure.

The wake-up call theory of contagion has two testable implications. The first implications is that the strength of the wake-up call contagion increases in the extent of ex-ante exposure across regions. This implication is consistent with empirical evidence on wake-up call contagion. An empirical literature documents support for wake-up call contagion across markets and over time (e.g., Van Rijckeghem and Weder 2003; Karas et al. 2013; Giordano et al. 2013) and links the strength of wake-up call contagion to ex-ante exposure, such as institutional similarities. Dasgupta et al. (2011) document a positive association between the extent of ex-ante exposure to a ground zero crisis country and wake-up call contagion of currency attacks, using a measure for institutional similarities. Also consistent with our model are findings from empirical corporate finance that associate common weak legal institutions for corporate governance to more severe crises (Johnson et al. 2000; Mitton 2002). Moreover, consistent evidence is also offered in Karas et al. (2013) who document wake-up call contagion in banking.

The second testable implication is that information acquisition about the exposure to aggregate or market-wide shocks increases not only after observing a crisis elsewhere, but also in the extent of ex-ante exposure across regions. We are not aware of existing empirical evidence on the latter part and encourage future empirical work. In the main text, we discuss some directions for future empirical work in the contexts of currency attacks, debt crises, and bank runs.

We also explore some normative implications of the model. First, we show that "ignorance can be bliss" (Proposition 4), whereby a higher information cost reduces information acquisition by investors and can reduce the ex-ante probability of regime change, a measure of welfare relevant for several policymakers. Second, we connect to the literature on transparency (e.g., Morris and Shin (2002)) and show that greater transparency can actually crowd in private information production (Proposition 5). That is, more precise public information can induce more information acquisition.

The wake-up call theory of contagion can be informative for a range of economic applications. For currency crises, speculators observe a currency attack and are uncertain about the magnitude of trade or financial links or institutional similarity.[1] For rollover risk and bank runs, wholesale investors observe a run elsewhere and are uncertain about potential interbank exposures and insti-

[1]See Morris and Shin (1998) and Corsetti et al. (2004) for a one-regional global game that builds on the earlier works of Krugman (1979), Flood and Garber (1984), and Obstfeld (1986). For trade links, financial links, and institutional similarities see Glick and Rose (1999), Van Rijckeghem and Weder (2001, 2003), and Dasgupta et al. (2011).

tutional and regulatory similarities across banks.[2] For sovereign debt crises, bond holders observe a sovereign default elsewhere and are uncertain about the macroeconomic links, the commitment of the international lender of last resort, or the resources of multilateral bail-out funds.[3] For political regime change, activists observe a revolution, for example during the Arab spring, and are uncertain about the impact on their government's ability to stay in power.[4]

A key assumption for the result on the differential information choice of investors is the negatively skewed macro shock. An extensive empirical literature shows the negative skewness of important macroeconomic variables, including GDP growth, individual stock returns, and the aggregate stock market. The literature on asymmetric business cycles studies the occurrence of sharp recessions and slow booms (e.g., Neftçi 1984).[5] Barro (2006) links the high equity premium to the occurrence of rare disasters. More recent empirical research highlights the growth vulnerability dynamics, focusing on downside risks.[6] The negative skewness also plays a prominent role in the context of financial liberalization in developing countries, where changes to institutional and development models can promote growth but also create vulnerabilities.[7]

Our model aims to isolate the wake-up call component in the transmission of financial crises. Therefore, we abstract from other contagion channels explored in the literature. For contagion due to balance sheet links, see Allen and Gale (2000), Kiyotaki and Moore (2002), and Dasgupta (2004). For a common discount factor channel, see Ammer and Mei (1996) and Kodres and Pritsker (2002). Regarding a common investor base, see Goldstein and Pauzner (2004) for wealth effects, Pavlova and Rigobon (2008) for portfolio constraints, Taketa (2004) and Oh (2013) for learning about other investors. In terms of ex-post exposures, see Basu (1998) for a common risk factor, and Chen (1999), Acharya and Yorulmazer (2008), Manz (2010) and Allen et al. (2012) for asset commonality among banks and information contagion.

[2]See Rochet and Vives (2004) and Goldstein and Pauzner (2005) for one-region global-games bank run mdoels.

[3]See Corsetti et al. (2006). See Drazen (1999) for membership contagion.

[4]For a one-regional global game of political regime change with endogenous information manipulation or dissemination, see Edmond (2013) and Shadmehr and Bernhardt (2015), respectively.

[5]When measured over long periods, the negative skewness of real per capita GDP growth can be substantial, exceeding -5 for some countries (Barro 2006). Theoretical explanations for the negative skewness of output and total factor productivity include Acemoglu and Scott (1997), Veldkamp (2005), and Jovanovic (2006). Campbell and Hentschel (1992) and Bae et al. (2007) study the sources of the negative skewness of stock returns.

[6]For example, Adrian et al. (2019) study the evolution of the conditional distribution of future U.S. GDP growth and find that a build-up of negative skewness is associated with worsening financial conditions.

[7]Rancière et al. (2003) find that financially liberalized developing countries exhibit a negatively skewed growth of both GDP and credit. Rancière et al. (2008) argue that a negative skewness captures systemic risk.

Calvo and Mendoza (2000) and Mondria and Quintana-Domeque (2013) also have endogenous information. Contagion arises in Calvo and Mendoza since globalization shifts the incentives of risk-averse investors from costly information acquisition to imitation and herding. In Mondria and Quintana-Domeque, the contagion mechanism is based on the reallocation of limited attention by risk-averse investors, where a higher relative attention allocated to one market induces a higher price volatility in another market. In contrast, we highlight a complementary channel where a wake-up call induces information acquisition about a macro shock and contagion without common investors, risk-aversion, or information processing constraints.

Our modeling approach is closest to the literature on information choice in global coordination games initiated by Hellwig and Veldkamp (2009). They show that the information choices of investors inherit the strategic motive of an underlying beauty contest, which can result in multiple equilibria. Our game of regime change with complementarity in actions also yields strategic complementarity in information choices. While multiple equilibria exist, a sufficiently negatively skewed macro shock ensures a uniqueness for an intermediate range of information costs. In contrast to the acquisition of publicly available information, Szkup and Trevino (2015) and Ahnert and Kakhbod (2017) examine private information acquisition in global games of regime change.

Our theory is also related to the literature on financial crises that rationalizes how small shocks have large effects. Dang et al. (2015) and Gorton and Ordoñez (2014, 2019) study how information-insensitive debt can become information-sensitive if fundamentals deteriorate, triggering adverse selection concerns and information acquisition about the collateral backing the loans. As in our paper, information acquisition follows negative news but our channel differs as we analyze a Bayesian learning channel about macro fundamentals. Instead of a fear of adverse selection, there is a fear of macro downside risk with the skewed macro shock driving the differential information choice.

This paper proceeds as follows. Section 2 describes the model. We solve for its equilibrium and describe our contagion results in Section 3. Section 4 describes testable implications of the model and relates them to existing evidence as well as to how they may inform future empirical work. Section 5 derives normative implications of the model and discusses extensions and robustness issues. Section 6 concludes. All proofs are in the Appendix.

2 Model

We study global coordination games of regime change played sequentially in two regions, $t = 1,2$. Each region has a different unit continuum of risk-neutral investors $i \in [0,1]$. Investors in region $t = 1$ move first, followed by investors in region $t = 2$.

Attack decision. In each region, investors simultaneously decide whether to attack a regime, $a_{it} = 1$, or not, $a_{it} = 0$. The outcome of the attack depends on the aggregate attack size, $A_t \equiv \int_0^1 a_{it}\, di$, and a regional fundamental $\Theta_t \in \mathbb{R}$ that measures the strength of the regime. A regime change occurs if enough investors attack, $A_t > \Theta_t$. Following Vives (2005), an attacking investor in region t receives a benefit $b_t > 0$ if a regime change occurs and otherwise incurs a loss $\ell_t > 0$, where $\gamma_t \equiv \frac{\ell_t}{b_t+\ell_t} \in (0,1)$ is the relative cost of failure:

$$u(a_{it} = 1, A_t, \Theta_t) = b_t \;\; \mathbf{1}_{\{A_t > \Theta_t\}} - \ell_t \;\; \mathbf{1}_{\{A_t \leq \Theta_t\}}. \tag{1}$$

The payoff from not attacking is normalized to zero, so the relative payoff from attacking increases in the attack size A_t (global strategic complementarity in attack decisions) and decreases in Θ_t.

Examples of a regime change include a currency attack, bank run, or debt crisis. The fundamental is interpreted as the ability of a monetary authority to defend its currency (Morris and Shin 1998; Corsetti et al. 2004), the measure of investment profitability (Rochet and Vives 2004; Goldstein and Pauzner 2005; Corsetti et al. 2006), or the ability or willingness of a debtor to repay. Investors are interpreted as currency speculators who attack a currency peg, as (uninsured) retail or wholesale bank creditors who withdraw funds, or as debt holders who refuse to roll over.

Macro shock. Each regional fundamental Θ_t comprises a regional component θ_t and a common component m. This common macro shock is the only link between regions:

$$\Theta_t = \theta_t + m, \tag{2}$$

where each θ_t follows an independent normal distribution with mean $\mu \in (-\infty, \infty)$ and precision $\alpha_t \in (0, \infty)$ and θ_t is independent of the macro shock. Unless stated otherwise, we consider $\alpha_1 = \alpha_2 \equiv \alpha$. Depending on its realization, the macro shock induces a positive correlation between regional fundamentals Θ_1 and Θ_2. Specifically, region 2 is ex-post exposed to region 1 via the common macro shock if $m \neq 0$.[8] The macro shock is assumed to take one of three values:

$$m = \begin{cases} \Delta & \quad p \\ -s\Delta \quad \text{w.p.} & \quad q \\ 0 & \quad 1-p-q, \end{cases} \tag{3}$$

where $p \in [0,1]$, $q \in [0,1-p]$, $\Delta > 0$, $s > 0$. We impose $p = qs$ to ensure an unbiased macro shock. Its variance is $p(1+s)\Delta^2$ and its skewness is $\frac{1-s}{\sqrt{p(1+s)}}$, which is negative if and only if $s > 1$.[9]

The macro shock is initially unobserved, motivated by our applications to financial crises. For currency attacks or sovereign debt crises, this uncertainty about the macro shock reflects the unknown relevance of certain institutional similarities, common vulnerabilities, or linkages across debtors. For bank runs, it reflects potential interbank exposures, common institutional weaknesses in governance, regulation and financial (mis-)conduct that gives rise to vulnerabilities.

Incomplete information. Following Carlsson and van Damme (1993), there is incomplete information about the fundamental. Each investor receives a noisy private signal x_{it} before deciding whether to attack (Morris and Shin 2003):

$$x_{it} \equiv \Theta_t + \varepsilon_{it}. \tag{4}$$

Idiosyncratic noise ε_{it} is identically and independently normally distributed with zero mean and precision $\beta \in (0, \infty)$. Each noise term is independent of the macro shock and regional component.

Information acquisition. Our setup features an information stage that precedes a coordination stage in region 2 (see timeline in Table 1). First, investors in region 2 observe whether there is a crisis in region 1. Second, investors can acquire costly information about the macro shock.[10] Thereafter, investors simultaneously decide in the coordination stage whether to purchase a per-

[8] For an alternative interpretation in which the exposure of region 2 to the macro shock and the macro shock realization are disentangled, see Section 5.4.

[9] The parameter s also affects the variance of the macro shock but its effect on the skewness is key. In Section 5.2, we argue that the negative skewness governed by s is at the heart of our results and study modifications to our model, including changes to Δ such that s only affects the skewness of the macro shock.

[10] We abstract from information acquisition in region 1 without loss of generality (see also Section 5.5).

fectly revealing signal about the macro shock at cost $c > 0$.[11] In terms of wholesale investors or currency speculators, costly information acquisition could be the hiring of analysts who assess publicly available information to gauge the relevance of institutional characteristics (such as structural or policy distortions, weak governance, etc.) and potential vulnerabilities that are shared across regions or banks and make them prone to changes in the macroeconomic or financial environment.

Date 1	**Date 2**
1. Macro shock m and regional component θ_1 realized but unobserved	1. Regional component θ_2 realized but unobserved
	Information stage in region 2
Coordination stage in region 1	
	2. Investors choose whether to acquire information about macro shock m at cost $c > 0$
2. Investors receive private information x_{i1} and choose whether to attack the regime, $a_{i1} \in \{0,1\}$	
	Coordination stage in region 2
3. Payoffs to investors in region 1	
	3. Investors receive private information x_{i2} and choose whether to attack the regime, $a_{i2} \in \{0,1\}$
4. Outcome of regime publicly observed	
	4. Payoffs to investors in region 2

Table 1: Timeline of events.

3 Equilibrium

3.1 Region 1

We first consider the equilibrium in region 1. A Bayesian equilibrium is an attack decision a_{i1} for each investor i and an aggregate attack size A_1 that satisfy both individual optimality for all investors, $a_{i1}^* = \arg\max_{a_{i1} \in \{0,1\}} \mathbb{E}[u(a_{i1}, A_1, \Theta_1)|x_{i1}]$, and aggregation, $A_1^* = \int_0^1 a_{i1}^* di$. Let $n_1 \in [0,1]$ be the proportion of investors in region 1 informed about the macro shock. If all are informed, $n_1 = 1$, the analysis is standard (see, e.g., Morris and Shin (2003, 2004)). If some investors are uninformed, $n_1 < 1$, the analysis is non-standard and requires the use of mixture distributions.

[11] We discuss an extension to noisy signals about the macro shock in Section 5.5.

We focus on the case of uninformed investors, $n_1 = 0$, but the result can be readily extended to $n_1 \in (0,1)$ following the same steps as in the analysis of the coordination stage in region 2.

Lemma 1 ***Equilibrium in region 1.*** *Let $n_1 = 0$. If private information is sufficiently precise, there exists a unique monotone Bayesian equilibrium. Each investor attacks when the private signal is below a signal threshold, $x_{i1} < x_1^*$. A crisis occurs when the fundamental is below a fundamental threshold, $\Theta_1 < \Theta_1^*$.*

Proof See Appendix A.1.

Lemma 1 extends the analysis in standard global games models (e.g. Morris and Shin (2003)) to the case where the posterior of investors follows a mixture distribution over different macro states, comprising conditional normal distributions. The equilibrium is characterized by an indifference condition from individual optimality and by a critical mass condition which states that the proportion of attacking investors A_1^* equals the fundamental threshold Θ_1^*. The equilibrium conditions can be reduced to one equation in one unknown. Using the results of Milgrom (1981) and Vives (2005), the best-response function of individual investors are strictly increasing in the thresholds used by other investors (Appendix A.1.1). The common requirement of sufficiently precise private information suffices for uniqueness in monotone equilibrium in the case of mixture distributions.

3.2 Region 2

Considering region 2, let $n_2 \in [0,1]$ be the proportion of investors in region 2 who acquire information about the macro shock m and $d_i \in \{I,U\}$ is the information choice of investor i, with corresponding attack rules of informed and uninformed investors, $a_I \equiv a_{i2}(d_i = I)$ and $a_U \equiv a_{i2}(d_i = U)$.

Definition 1 *A pure-strategy monotone perfect Bayesian equilibrium in region 2 comprises an information choice for each investor, $d_i^* \in \{I,U\}$, an aggregate proportion of informed investors, $n_2^* \in [0,1]$, an attack rule for informed and uninformed investors, $a_I^*(m,\cdot)$ and $a_U^*(\cdot)$, and an aggregate attack size, A_2^*, such that:*

1. *At the information stage, investors optimally choose their information d_i.*

2. *The proportion of informed investors is consistent with individual choices, $n_2^* = \int_0^1 d_i^*\, di$.*

3. *At the coordination stage, attack rules are optimal, where uninformed investors use $a_U^*(\cdot)$ and informed investors use $a_I^*(m,\cdot)$ for each macro shock.*

4. *The aggregate attack size is consistent with attack rules for each macro shock:*

$$A_2^* \quad = \quad n_2^* \int_0^1 a_I^*(m,\cdot)di + (1-n_2^*)\int_0^1 a_U^*(\cdot)di. \tag{5}$$

To derive analytical results, we maintain the following assumption throughout.

Assumption 1 *Private information is precise, $\beta > \underline{\beta}$, public information is imprecise, $\alpha < \overline{\alpha}$, a zero macro shock is unlikely, $1-p-q<\eta$, crisis are rare, $\mu > \underline{\mu}$, the macro shock is sufficiently negatively skewed, $s > \underline{s} \geq 1$.*

Assumption 1 states sufficient conditions for the main result on wake-up call contagion, where the bounds are described in the proofs. The rareness of crises implies a strong fundamental re-assessment after the wake-up call of a crisis in region 1 and the negative skewness is crucial for the incentives of investors to acquire information only after observing a crisis but not for the Bayesian updating channel.[12] The assumption of a sufficiently high relative precision of private information is common in the global games literature (e.g. Vives 2005). While sufficiently imprecise public information is not required for the existence of unique attack rules, it leads to concentrated posterior beliefs about the macro shock and facilitates the analysis of how equilibrium fundamental and signal thresholds vary with the proportion of informed investors. Lastly, the sufficiently low probability of a zero macro shock simplifies the analysis. It allows us to focus on the favorable and unfavorable macro states central to the re-assessment. These conditions are sufficient, but not necessary, and help with tractability and exposition. The numerical examples below show that our results also obtain under less restrictive conditions. We further discuss the robustness in Section 5.

We proceed by constructing the equilibrium in region 2. Investors in region 2 observe whether a crisis occurred in region 1, and use Bayes' rule to re-assess the fundamental of region 2, specifically the macro shock m. Since only a proportion of investors may choose to acquire information, we allow for heterogeneous priors. There are three distinct fundamental thresholds – one for

[12] For a discussion of positive skewness, $s \in (0,1)$, see Section 5.3.

each realized macro shock – and thus three critical mass conditions. Similarly, there are four indifference conditions – one for uninformed investors and one for informed investors for each macro shock realization. The system of equations is derived in Appendix A.2. If some investors are informed, we denote the fundamental thresholds in region 2 as $\Theta_I^*(m)$. If all investors are uninformed, $n_2 = 0$, we denote the fundamental thresholds in region 2 as Θ_U^*.

Proposition 1 ***Equilibrium in region 2.*** *For intermediate information costs, $c \in (\underline{c}, \overline{c})$, there exists a unique monotone perfect Bayesian equilibrium. At the information stage, investors acquire information only after a wake-up call, $n_2^* = \mathbf{1}_{\{\Theta_1 < \Theta_1^*\}}$. At the coordination stage, investors use threshold strategies:*

1. *After no crisis in region 1, investors choose to be uninformed and attack whenever their private signal is sufficiently low, $x_{i2} < x_U^*$, and a crisis occurs whenever the fundamental is sufficiently low, $\Theta_2 < \Theta_U^*$.*

2. *After a crisis in region 1, investors choose to be informed and attack whenever their private signal is sufficiently low relative to a macro-shock-specific threshold $x_{i2} < x_I^*(m)$, and a crisis occurs whenever the fundamental is sufficiently low relative to a macro-shock-specific threshold, $\Theta_2 < \Theta_I^*(m)$.*

Proof See Appendix A.2.5 for a proof and Appendix A.2 for a derivation of the equilibrium conditions, as well as the formal statements on information acquisition discussed below.

The equilibrium is in dominant actions at the information stage. Irrespective of the information choices of other investors, each investor acquires information *only* after the wake-up call of a crisis in region 1. This occurs whenever the fundamental in region 1 is below its threshold, $\Theta_1 < \Theta_1^*$. When investors in region 2 choose to be uninformed, they use the same attack threshold, x_U^*, and there is one fundamental threshold, Θ_U^*, where both thresholds are independent of the macro shock. In contrast, when investors choose to be informed, they tailor their attack rule to the macro state, $x_I^*(m)$, and there is one fundamental threshold for each state, $\Theta_I^*(m)$.

We next build intuition for the differential information choice in Proposition 1. Examining the value of information, we trace out how investors' incentives to acquire information about the macro shock are affected by the wake-up call and other investors' information choices. Let $f \in \{0,1\}$

indicate whether a crisis occurred in region 1. After a wake-up call, $f = 1$, investors learn that the fundamental in region 1 was low, $\Theta_1 < \Theta_1^*$. Conversely for $f = 0$, the fundamental was high, $\Theta_1 \geq \Theta_1^*$. Using Bayesian updating, Lemma 2 in Appendix A.2.1 states the intuitive result that a less (more) favorable macro shock realization is more likely after a crisis (no crisis).

The resulting re-assessment determines the incentives of investors to acquire information, with a higher value of information after a wake-up call. Since crises are rare events, there is a strong Bayesian updating channel after a wake-up call. The negatively skewed macro shock generates an asymmetry, which assures that it is more valuable to acquire information about the realized macro shock after observing a crisis in region 1. The probability of a negative macro shock is small without a crisis in region 1, but it is substantially higher after a crisis. Hence, investors in region 2 have a high benefit from learning about the macro shock and tailoring their attack decision. This is the key effect behind the differential information choice in Proposition 1.

We proceed by discussing the value of information and how it affects the information acquisition incentives. The value of information is defined as the difference between the expected utility of an informed investor, EU_I, and an uninformed investor, EU_U, as derived in Appendix A.2.3. It depends on the proportion of informed investors and on whether a crisis occurred in region 1:

$$v(n_2, f) \equiv EU_I - EU_U. \tag{6}$$

Informed investors observe whether a crisis occurred and take into account the possible realizations of m, since these affect the signal thresholds, $x_I^*(m)$. By contrast, uninformed investors cannot tailor their attack strategy and must use the *same* signal thresholds x_U^* for all realized macro shocks. As a result, the signal thresholds of informed and uninformed investors differ and $v(n_2, f) > 0$.[13]

Information about the macro shock allows an investor to tailor her behavior and reduce two types of errors. First, when an investor attacks the regime although no crisis occurs, she incurs a loss (type-I error). Second, when an investor does not attack although a crisis occurs, she could have earned a benefit (type-II error). The value of information is governed by the relationship between

[13] To evaluate the incentives of investors to acquire information, we study the optimal attack behavior for any given proportion of informed investors and allow for some investors to be informed while others are uninformed, resulting in heterogeneous priors about the macro shock that follow a mixture distribution. In another global game with mixture distributions, Chen et al. (2016) develop a theory of rumors during political regime change. However, they abstract from both contagion and information choice.

these two types of errors. The marginal benefit of increasing $x_I^*(-s\Delta)$ above x_U^* is positive because the type-II error is relatively more costly than the type-I error. By contrast, the marginal benefit of decreasing $x_I^*(\Delta)$ below x_U^* is positive because the type-I error is more costly. In sum, informed investors attack more aggressively upon learning the low macro shock realization, $m = -s\Delta$, and less aggressively upon learning the high realization, $m = \Delta$.

Next, we turn to the strategic aspect of information acquisition. The signal thresholds of informed and uninformed investors depend on the proportion of informed investors. We find that the difference in signal thresholds increases monotonically in the proportion of informed investors, as derived in Lemma 3 in Appendix A.2.2. The divergence of signal thresholds with an increasing proportion of informed investors induces a strategic complementarity in information choice, $\frac{dv(n_2,f)}{dn_2} \geq 0$, as derived in Lemma 4 in Appendix A.2.3. Intuitively, the individual attack decision of an informed investor is more strongly adjusted the larger the proportion of informed investors, which in turn increases the value of information. In the words of Hellwig and Veldkamp (2009), investors want to know what others know in order to do what others do.

Figure 1 shows the attack threshold of informed and uninformed investors. First, informed investors attack more (less) aggressively after observing a negative (positive) macro shock. Second, conditional on observing a wake-up call, both types of investors attack more aggressively. Comparing the left and right panel, all signal thresholds are higher after a wake-up call for each macro state and for all $n_2 \in [0,1)$. When all investors are informed, $n_2 = 1$, the signal thresholds coincide irrespective of whether a crisis occurred in region 1, since region 1's outcome does not contain any information beyond the macro shock.

The relationship between the signal thresholds of informed investors is derived in Lemma 3 in Appendix A.2.2. Strict divergence of $x_I^*(n_2,-s\Delta)$ and $x_I^*(n_2,\Delta)$ in the proportion of informed investors follows from $\frac{d\Theta_2^*(n_2,-s\Delta)}{dn_2} > 0$ and $\frac{d\Theta_2^*(n_2,\Delta)}{dn_2} < 0$. Lemma 3 shows that the signal thresholds are monotonic in the proportion of informed investors. Moreover, $x_I^*(n_2,0)$ and $x_U^*(n_2,m)$ are bounded by $x_I^*(1,-s\Delta)$ and $x_I^*(1,\Delta)$. Solving the equilibrium condition in equation (26) when all investors are uninformed, we find that $[\Theta_U^*|f=1] > [\Theta_U^*|f=0]$ and, hence, $[x_I^*(0,m)|f=1] > [x_I^*(0,m)|f=0]$ for all $m \in (-s\Delta, 0, \Delta)$. The right panel of Figure 1 shows this upward shift in signal thresholds for all $n_2 < 1$, which stems from the updating of uninformed investors' belief

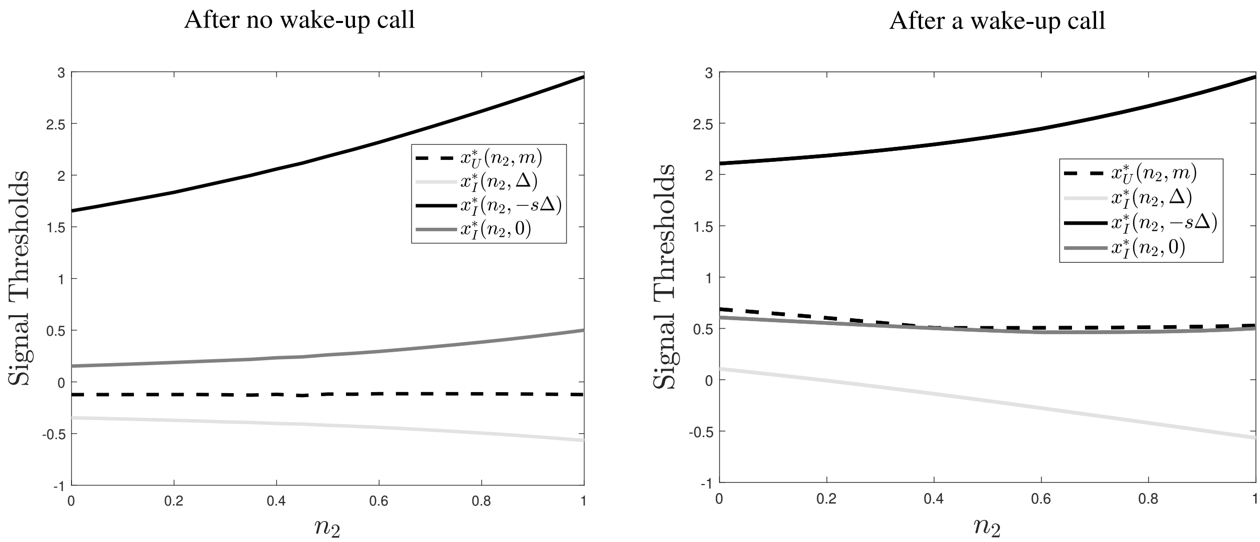

Figure 1: Signal thresholds of informed and uninformed investors, x_I^* and x_U^*, as a function of the proportion of informed investors, n_2, after observing no crisis in region 1 (left panel) and a crisis (right panel), as characterized in Lemma 3. Parameter values are $\alpha = \beta = 1$, $\mu = \Delta = \gamma = p = 1/2$ and $s = 3$.

about the macro shock (Lemma 2). The shift is stronger if more investors are uninformed, but the difference in thresholds is already noticeable for $n_2 = 0$: $[x_U^*(0,m)|f = 1] > \frac{1}{2} > [x_U^*(0,m)|f = 0]$.

Building on these insights, Proposition 2 ranks the value of information after a wake-up call and after no wake-up call when all investors make the same choices, $n_2 \in \{0,1\}$.

Proposition 2 ***Wake-up call and the value of information.*** *The value of information is higher after a crisis in region* 1 *independent of the proportion of informed investors:*

$$v(1,1) > v(0,1) > v(1,0) > v(0,0). \tag{7}$$

Proof See Appendix A.2.4.

The first and third inequality in (7) represent the strategic complementarity in information choices derived in Lemma 4 in Appendix A.2.3. The second inequality is due to the negatively skewed macro shock. For a sufficiently negatively skewed macro shock and rare crisis, as guaranteed by Assumption 1, we have $v(0,1) > v(1,0)$. As a result, there exists an intermediate range of information costs $c \in (\underline{c}, \overline{c})$ with $\underline{c} \equiv v(1,0)$ and $\overline{c} \equiv v(0,1)$ such that all investors choose to acquire information if and only if a crisis occurs in region 1 (the wake-up call).

Figure 2 offers an illustration. The left panel shows that the value of information increases

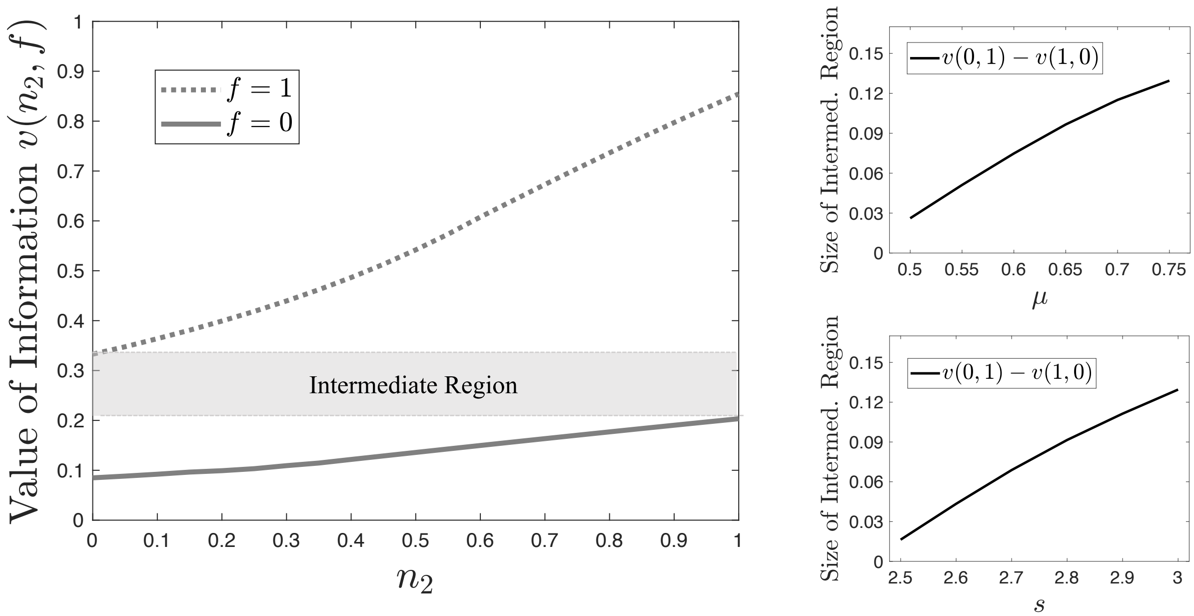

Figure 2: The left panel depicts the value of information v and the proportion of informed investors n_2 with a wake-up call ($f = 1$) and without a wake-up call ($f = 0$). The shaded area shows the intermediate range of information costs, $(\underline{c}, \overline{c})$, for $s = 3$ and $\mu = 3/4$. The top right panel depicts the size of the intermediate region, $v(0,1) - v(1,0)$, as μ increases from $1/2$ to $3/4$ (for $s = 3$). The bottom right panel shows the effect of a higher level of s that increases the negative skewness of the macro shock (for $\mu = 3/4$). The other parameters are as in Figure 1. Formal statements can be found in Proposition 2 and Lemma 4.

in the proportion of informed investors due to a strategic complementarity (Lemma 4) and in the occurrence of a crisis in region 1 (Proposition 2). There exists an intermediate region (shaded area) where $v(0,1) > v(1,0)$. When crises are rare and the macro shock is sufficiently negatively skewed, the Bayesian updating channel is strong and ensures a unique equilibrium for intermediate values of information costs despite strategic complementarity in information choices.

While we established the existence of the intermediate region analytically, comparative statics are difficult to obtain in general. First, there is a tendency for the intermediate region to expand if crisis are less frequent (for higher μ). This is shown in the top right panel of Figure 2, where we start from $\mu = 1/2$ which implies a relatively high crisis incidence (see also Figure 3). Second, the intermediate region also expands if s increases, as shown in the bottom right panel of Figure 2. The illustration is consistent with Proposition 2. A higher μ strengthens the Bayesian updating channel and a higher s increases the benefits from tailoring of signal thresholds, $\frac{d(x_I^*(1,-s\Delta) - x_I^*(1,\Delta))}{ds} > 0$.[14]

We proceed by describing the contagion mechanism and build intuition for the Bayesian updat-

[14] In the Online Appendix A.9, we analytically show for a special case that the differential value of information increases in the parameter s, governing the negative skewness of the macro shock, $\frac{d[v(1,1) - v(1,0)]}{ds} > 0$.

ing channel after the occurrence of a crisis in region 1.

3.3 Contagion

Having established a unique equilibrium for intermediate information costs, we turn to the question of contagion after a wake-up call. Contagion is defined as the increase in the likelihood of a crisis in region 2 after a crisis in region 1, compared to no crisis in region 1. Our main result is that contagion occurs even if investors learn that the macro shock is zero, meaning that region 2 has no ex-post exposure to region 1. This result isolates the wake-up call component of contagion and builds on the equilibrium information choices in Proposition 1 and holds under Assumption 1.

Proposition 3 ***Wake-up call contagion.*** *Let $c \in (\underline{c}, \overline{c})$. A financial crisis in region 2 is more likely after a crisis in region 1 when all investors acquire information and learn that the macro shock is zero, than after no crisis in region 1, when all investors choose not to acquire information:*

$$\Pr\{\Theta_2 < \Theta_I^*(m) | m = 0\} > \Pr\{\Theta_2 < \Theta_U^*\}. \tag{8}$$

Proof See Appendix A.3.

Proposition 3 rests on the unique equilibrium for intermediate information costs. The left-hand side of inequality (8) is the probability of a crisis in region 2 after a crisis in region 1, a wake-up call, that induces investors *to acquire information* and when they learn that the macro shock is zero. The right-hand side is the probability of a crisis in region 2 after no crisis in region 1, that induces investors *not to acquire information*. Hence, the conditional probability implicit in the right-hand side allows for any realization of the unobserved macro shock.

We find that a crisis in region 2 is more likely after a crisis in region 1 than after no crisis in region 1 *even if* all investors acquire information and learn that the macro shock is zero. Learning that the macro shock is zero implies that region 2 is not ex-post exposed to (the crisis in) region 1. In contrast, no crisis in region 1 implies a more favorable view about the fundamental in region 2 due to the unobserved macro shock. Hence, the decreased crisis probability after observing no crisis in region 1 is a key driver of the result. This effect tends to lower the right-hand side of inequality (8). Thereby, Proposition 3 isolates the wake-up call component of contagion by showing that contagion occurs even if investors learn that the macro shock is zero.

The contagion result is further strengthened by noting that the probability of a crisis in region 2 when all investors acquire information and learn that the macro shock is zero, $\Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\}$, is also higher than the *ex-ante* probability of a crisis in region 2 absent the learning of a state about region 1. We formally present and prove this additional result in Appendix A.4.[15]

The Bayesian updating channel underlying the result in Proposition 3 builds on the re-assessment of the macro shock. Intuitively, the observation of a crisis in region 1 can whip around probabilities of tail events and focus attention on rarely observed downside risks. That is, after *observing a crisis* in region 1, the conditional expectation about the macro shock is negative, $E[m|f = 1] < 0$. This result is due to an upward revision of the probability of a negative macro shock after bad news, $\Pr\{m = -s\Delta|f = 1\} > q$. After *observing no crisis* in region 1, by contrast, the expectation about the macro shock is positive, $E[m|f = 0] > 0$, since the probability of a negative macro shock is revised down. The re-assessment is stronger—that is the difference between $E[m|f = 1]$ and $E[m|f = 0]$ is higher—when the average strength of regional fundamentals is higher and crises are rare. Specifically, an increase in μ or in s increases the magnitude of the re-assessment for the same reasons. Moreover, the Bayesian updating results arise even if $s = 1$ (no skewness).

While Bayesian updating is fairly mechanical, we note that the result of wake-up call contagion arises endogenously. For intermediate information costs, investors choose to acquire information only after the wake-up call. In other words, the comparison of scenarios in equation (8) hinges on the differential information choice. Note that the assumption of a negatively skewed macro shock is inessential for the Bayesian updating channel that governs inequality (8). However, it is crucial for the information choice underlying the comparison in inequality (8), which is driven by the strong fundamental reassessment. Hence, the wake-up call effect relies on downside risk in the form of rare but strongly negative shocks to fundamentals.

Figure 3 illustrates the magnitude of the wake-up call contagion effect. We compare the crisis probability by plotting both sides of inequality (8). In the numerical example, the effect is significant and its magnitude can exceed 15%.[16] The magnitude is governed by the strength of the

[15] We thank an anonymous referee for pointing out this result.

[16] The bounds on the precision of private and public signals are not more stringent than the standard conditions used

Bayesian updating channel, which hinges on the ex-ante probability of a zero macro shock. If this probability is higher—i.e., if the ex-ante exposure of region 2 is reduced—then the wake-up call contagion channel weakens. Intuitively, absent a crisis in region 1, uninformed investors place a higher probability $\Pr\{m = \Delta | f = 0\} > p$ on a positive realization of the macro shock. Given that an increase in the probability of a zero macro shock implies a reduction in p, it is associated with a less favorable view about fundamentals after not observing a crisis. Taken together, the difference in likelihoods of a crisis in region 2 from inequality (8) is positive and decreasing in $1 - p - q$. This effect speaks to the empirical literature and we develop the result formally in Section 4 below.

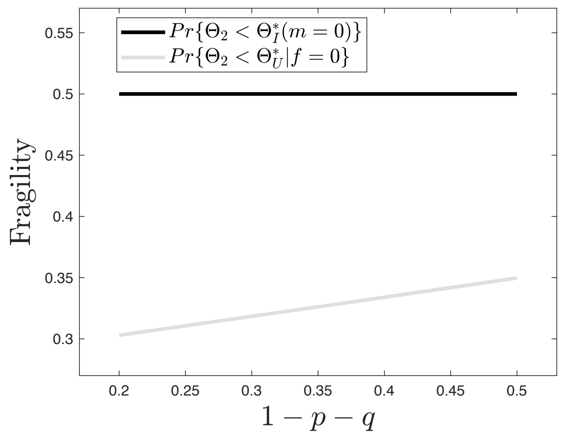

Figure 3: The magnitude of the wake-up call contagion effect in isolation, measured as the difference between the two sides of inequality (8) in Proposition 3, decreases if the ex-ante exposure of region 2 is reduced when the probability of a zero macro shock, $1 - p - q$, increases from 0.2 to 0.5. The other parameter values are as in Figure 1.

4 Testable Implications

The wake-up call theory of contagion has two testable implications described in this section. We discuss how the first implication is consistent with existing evidence and suggest avenues for testing the second implication in future empirical work.

Implication 1: *After controlling for direct links, the strength of the wake-up call contagion channel increases in the extent of ex-ante exposure across regions.*

to assure uniqueness of equilibria in global games models (Morris and Shin 2003; Svensson 2006). Notably, the result of wake-up call contagion prevails when the probability of a zero macro shock is rather high (such as $1 - p - q = 1/3$) and when crises are relatively frequent (such as $\mu = 1/2$), illustrating the robustness of the key results from relaxing the sufficient conditions stated in Assumption 1.

The formal result underlying Implication 1 is illustrated in Figure 3 and derived in a corollary to Proposition 3 stated below. As explained in Section 3.3, the intuition hinges on the fact that an increase in the probability of a zero macro shock (i.e., a decrease in ex-ante exposure) is associated with a less favorable view about fundamentals after not observing a crisis. Corollary 1 presents the formalization and we note that the analytical result extends beyond the special case of symmetry ($\mu = \gamma = \frac{1}{2}$ and $s = 1$), as is shown in Figure 3.

Corollary 1 ***Magnitude of wake-up call contagion.*** *If $\mu = \gamma = \frac{1}{2}$, $s = 1$, and sufficiently low p and α_1, then the magnitude of wake-up call contagion increases in the degree of ex-ante exposure:*

$$\frac{d \Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\} - \Pr\{\Theta_2 < \Theta_U^*\}}{d(1 - p - q)} < 0. \tag{9}$$

Proof See Appendix A.5.

The first testable implication is consistent with empirical evidence on wake-up call contagion.[17] An empirical literature documents support for wake-up call contagion across markets and over time (e.g., Van Rijckeghem and Weder 2003; Karas et al. 2013; Giordano et al. 2013) and links the strength of wake-up call contagion to ex-ante exposure (Dasgupta et al. 2011). A key source of ex-ante exposure are institutional similarities. Dasgupta et al. (2011) document a positive association between the extent of ex-ante exposure to a ground zero crisis country and wake-up call contagion, using a measure for institutional similarities. This finding supports Implication 1 of our model. Also consistent with our model are findings from empirical corporate finance that associate common weak legal institutions for corporate governance to more severe crises (Johnson et al. 2000; Mitton 2002). Consistent evidence is also offered in Karas et al. (2013) who document wake-up call contagion in banking, emphasizing the institutional credibility of deposit insurance, which can relate to (unexpected changes in) design features and sovereign risk (Bonfim and Santos 2020).

The second testable implication concerns information acquisition, a key ingredient of the wake-up call contagion channel. The demand for information is a research area that has attracted increasing attention in the recent empirical literature (see, e.g., Da et al. (2011)). Proponents of this

[17]Empirically, wake-up call contagion may be accompanied by other contagion channels, such as balance sheet links, trade links, or portfolio effects via common investors (Forbes 2012).

literature find a positive correlation between the demand for information for aggregate or market-wide shocks and measures of volatility (Vlastakis and Markellos 2012). This is consistent with our theory, which links the information acquisition to the observation of a financial crisis elsewhere (Implication 2a). What is more, we can also offer guidance to future empirical work by linking the extent of information acquisition to the extent of ex-ante exposure (Implication 2b).

Implication 2: *The extent of information acquisition about the exposure to aggregate or market-wide shocks is (a) higher after observing a financial crisis elsewhere than after observing no crisis and (b) increasing in the extent of ex-ante exposure across regions.*

Implication 2a stems from equation (7) in Proposition 2 and Implication 2b is derived below as a corollary to Proposition 2.

Corollary 2 ***Value of information and ex-ante exposure.*** *For sufficiently low α_1 and Δ:*

$$\frac{dv(0,1)}{d(1-p-q)} < 0. \tag{10}$$

Proof See Appendix A.6.

Corollary 2 states that the value of information after observing a crisis in region 1 decreases in the probability of a zero macro shock. The increase in the value of information in turn implies an increase in information acquisition for a given cost. Intuitively, the result derives from forces closely related to Corollary 1 since the stronger Bayesian updating also fuels the information acquisition incentives. It suffices to focus on the case of $n_2 = 0$ due to the strategic complementarity in information choices. The result is derived under Assumption 1 and the additional sufficient condition that α_1 and Δ are low. Notably, the result also holds for the parameters used in the previous figures, suggesting that it is not confined to the additional conditions used to simplify the analysis.

Discussion of future empirical work. Implications 2a and 2b could, for instance, be taken to the data in the context of currency and sovereign debt crises. Our theory predicts a larger extent of information acquisition about common macro risks that may stem from pre-existing institutional similarities to a crisis country and that are increasing in the extent of these similarities. Following the literature on the demand for information this prediction could be tested with suitable measures for information acquisition (such as the internet search intensity for macro news that relate to potential common vulnerabilities) or with the help of more indirect measures (such as the shift in attention in domestic news). The extent of pre-existing institutional similarities can be captured by indicators used in the literature that are based on the quality of governance, corporate disclosure, corruption, regulatory quality and the rule of law. Moreover, recent advances in natural language processing and new data may hold promise in measuring various dimensions of institutional similarities, such as aspects related to the quality in financial reporting (Dyer et al. 2017) or international regulatory and policy coordination (Armelius et al. 2020).

Another application is corporate debt markets. Consider a firm with publicly traded debt to be rolled over by investors. A crisis elsewhere refers to a spike in the credit risk of other firms in the same industry sector that may be associated with a substantial ratings downgrade or earnings warning. It is well know that institutional lenders like banks seek to insure against industry-specific risks when confronted with a significant exposure via portfolio trading or the loan book. Our theory predicts a high sensitivity of debt holders to negative news that may convey information about changes in industry-specific factors (e.g. demand factors, new trends, or innovations), as well as an increase in the incentive to acquire and analyze information about potential industry shocks and vulnerabilities of firms with certain institutional attributes and business models.

Finally, the testable implications of the wake-up call theory of contagion are also relevant for bank commercial paper, which is rolled over frequently. Apart from a downgrade or an earnings warning, a crisis elsewhere could also be a downward revision of another bank's asset quality by the supervisor or the discovery of financial misconduct that may bear relevance for other institutions with potentially similar shortcomings in corporate governance. Since macro variables appear to have a large effect on bank credit losses (see, for instance, Buncic et al. (2019)) and financial misconduct is often related to certain products, such a downgrade of a bank or supervisory action may trigger question not only regarding the direct exposure of other banks, but also regarding the role played by negative risk factors that are common across banks.

5 Additional results and discussion

We start with normative results on welfare and transparency (Section 5.1). We next probe the robustness of our results by considering several extensions and alternative modelling approaches. First, we discuss the negative skewness assumption and allow for a biased macro shock (Section 5.2). Then we look at the case of positive skewness (Section 5.3). Finally, we consider an alternative interpretation of exposure (Section 5.4) and several other robustness issues (Section 5.5).

5.1 Normative results

We discuss two measures: *(i)* utilitarian welfare and *(ii)* the ex-ante probability of regime change.

Utilitarian welfare is measured by the expected payoffs of investors. This measure is particularly relevant for the application of an investment game. For the parameters consistent with Assumption 1, the ex-ante utilitarian welfare weakly decreases in the information cost c. To see this, first observe that $v(n_2, f)$ is positive since individual investors can only gain from more information. Second, recall from Lemma 4 that there is a strategic complementarity in information choices, so it is beneficial for investors to become informed from both an individual and a social viewpoint. As a result, an increase in c has an unambiguously negative effect on utilitarian welfare.

The ex-ante probability of regime change is the second welfare measure considered. It is arguably a key variable of interest for a policymaker who wants to avoid a bank run or a currency attack. In general, the relationship between the information cost and ex-ante welfare is ambiguous and difficult to analyze. Specifically when comparing the scenario where investors acquire information after observing a crisis in region 1 with the scenario where they do not acquire information, the ex-ante probability of regime change can be higher or lower, depending on parameters.

Intuitively, the ranking depends on the relative weight of the differential crisis probabilities in region 2 by the conditional crisis probabilities in region 1. To see this, observe that the Bayesian updating channel is strong if the event of a rare crisis in region 1 is strongly linked with the occurrence of a negative macro shock so that more aggressive attacks by investors who acquire information and learn about a negative macro shock realization play a dominant part in the determination of the ex-ante probability of regime change.

Interestingly, we find that opacity can be good in our model. There are cases where the probability of regime change is higher if investors acquire information after a crisis in region 1 than if they never acquire information. To illustrate this point, we consider the special case of $\gamma = \mu = p = \frac{1}{2}$ (as in Figure 1 and 2) but also invoke stronger conditions than in Assumption 1.[18]

Proposition 4 ***Opacity can be bliss.*** *The ex-ante probability of regime change can be higher or lower for an intermediate information cost, $c \in (\underline{c}, \overline{c})$, than for a high cost, $c > \overline{c}$. An example for a higher ex-ante probability of regime change when investors acquire information arises for $\mu = \gamma = p = \frac{1}{2}$, $s = 1$ and a sufficiently small α_2.*

Proof See Appendix A.7.

This result is reminiscent of Dang et al. (2015) who show that ignorance can increase welfare. While we do not wish to draw a general policy recommendation from the special case analyzed in Proposition 4, it does show that a lower information costs can reduce a measure of welfare.

Next, we study how the incentives to acquire information are affected by transparency, measured by α_2 (e.g. Morris and Shin (2002)). Depending on the application, such an increase in the public signal precision can, for instance, be interpreted as an increase in market disclosure standards, the precision of information provided by rating agencies or as an increase in the transparency of bank stress tests. In the context of the debate about bank stress tests, higher transparency can be seen as a commitment of the banking regulator to disclose more detailed bank-specific information.

The general case is difficult to analyze analytically. We consider the special case of Proposition 4 and further simplify the analysis by considering circumstances in which the informativeness of learning about the crisis in region 1 vanishes such that the fundamental threshold in the case of uninformed investors is $1/2$. This property can be achieved if $\alpha_1 \to 0$, for example. Under the sufficient condition that Δ is high and α_2 is low, we find a positive association between transparency and the incentives to acquire information. Proposition 5 summarizes.

[18] The welfare analysis benefits from symmetry in this special case in which the fundamental (and signal) thresholds for the different macro shock states are equidistant for $s \to 1$: $\Theta_2^*(\Delta) - \Theta_2^*(0) = \Theta_2^*(0) - \Theta_2^*(-s\Delta)$. This limit case is not required for the equilibrium analysis or the results on information acquisition and wake-up call contagion.

Proposition 5 ***Transparency.*** *If $\mu = \gamma = p = \frac{1}{2}$, $s = 1$, sufficiently high Δ and sufficiently low α_1, α_2 , then greater transparency increases the incentives to acquire information:*

$$\frac{dv(1,f)}{d\alpha_2} > 0, \quad f \in \{0,1\}. \tag{11}$$

Proof See Appendix A.8.

Intuitively, higher incentives to acquire information with greater transparency arise from the larger benefit of tailoring the signal thresholds to the realized macro shock. An increase in transparency is associated with less aggressive attacks against the regime if the prior about the fundamentals is strong, which occurs if investors observe $m = \Delta$. At the same time, greater transparency is associated with more aggressive attacks against the regime if the prior about the fundamentals is weak, which occurs if investors observe $m = -s\Delta$. Hence, signal thresholds diverge. This effect is associated with an increase in the value of information and dominates for the case considered in Proposition 5, with opposing effects stemming from the curvature of the distribution functions.

The analytical result of Proposition 5 holds for the parameters used in previous figures. Importantly, it extends to larger values of s and to all $n_2 \in [0,1]$, suggesting that our result is not confined to the somewhat restrictive set of sufficient conditions stated in the proposition. The complementary relationship between disclosure and information acquisition established in Proposition 5 is, however, not a general result and we invoke stronger conditions as in Assumption 1.

While we cannot draw a general policy implication from the special case analyzed in Proposition 5, we can reject the view that more public disclosure inevitably reduces information acquisition. This observation contrasts with some of the literature that has analyzed the impact of transparency on information acquisition in coordination games. In the context of a beauty contests with private information acquisition, Colombo et al. (2014) find a crowding-out effect of public information; the incentives to acquire more precise private information decrease in the public signal precision.

In contrast, Szkup and Trevino (2015) study continuous information choice subject to a convex information cost that is homogeneous across investors. They analyze efficiency when information choices are complements or substitutes, and the trade-off between public and private information, focusing on the precision of public information. Ahnert and Kakhbod (2017) study binary private information choice subject to heterogeneous information costs, finding that greater disclosure sometimes increases fragility. In contrast, we study the acquisition of publicly available information in a regime change game. Finally, there is an earlier literature studying the effect of transparency on the incidence of a regime change with exogenous information (Morris and Shin 1998; Heinemann and Illing 2002; Bannier and Heinemann 2005).

5.2 Skewness of the macro shock

In this section, we further discuss the importance of the negative skewness of the macro shock as the key driver of the differential information choice (Proposition 2), which underpins our wake-up call contagion channel. Moreover, we show the robustness of our main results to two variations of our model. We first analyze a special case of the model with $s = 1$ to demonstrate that $s > 1$ is crucial for the differential information choice. Second, we consider a modified setup where we engineer offsetting changes of Δ that allow us to hold the variance of the macro shock constant when s changes. Third, we consider a setup where s and q can be varied independently. (In Section 5.3, we also discuss an alternative model setup with a positively skewed macro shock.)

We start with a special case of our model where $\mu = \gamma = \frac{1}{2}$, as in Figures 1–3. This simplifies the analysis and allows to discuss the role of the parameter s in a transparent way. For $s = 1$, the results in Lemma 3 continue to hold and we can show that the first and third inequality in Proposition 2 remain valid. However, the second inequality of Proposition 2 fails to hold because the value of information is identical in both scenarios when $s = 1$. The result is summarized in Corollary 3.

Corollary 3 *If $\mu = \gamma = \frac{1}{2}$ and $s = 1$, then $v(n_2, 0) = v(n_2, 1)$, $\forall n_2 \in [0, 1]$.*

Proof See Appendix A.10.1.

Corollary 3 highlights the role played by $s > 1$ for the differential information choice. Negative skewness drives a wedge between the relative incentives to acquire information, making information acquisition more valuable after observing a crisis. This leads to a strong Bayesian updating channel and whips around probabilities of tail events, focusing investor attention on downside risk. In sum, negative skewness is a critical element of our model of contagion.

Second, we study a version of the model with $\mu = \gamma = \frac{1}{2}$ in which changes in s are offset by changes in Δ in order to keep the variance of the macro shock constant at some $\chi > 0$ as s changes:

$$\Delta(s) \equiv \sqrt{\frac{\chi}{p(1+s)}} > 0, \qquad Var[m] = \chi.$$

Following an analogous argument as in Corollary 3, we find again no differential information choice if $s = 1$. Instead, under sufficient conditions akin to Assumption 1, the value of information is higher after observing a crisis, provided $s > 1$ is sufficiently high (see Appendix A.10.2).

Third, we consider the case when s and q can vary independently and the macro shock is biased. To be able to compare with our baseline model, we suppose that $p = q$. If $s = 1$, the argument in the proof of Corollary 3 is unchanged and we find that there is no differential information choice. For $s > 1$ the macro shock is biased, $E[m] < 0$. Under sufficient conditions akin to Assumption 1, inequality (7) of Proposition 2 continues to hold with the addition that the probability of the negative macro shock is sufficiently small. This is shown formally in Appendix A.10.3 where we also discuss the robustness of the wake-up call contagion result in Proposition 3, which holds for our baseline example but is now not guaranteed to hold more generally due to an additional effect.

5.3 Alternative model with positive skewness

Our model considers the empirically relevant case with a negatively skewed macro shock, which is key for our mechanism of wake-up call contagion based on endogenous information. While the observation of a crisis in region 1 can whip around probabilities of tail events and focus attention on rarely observed downside risk, information acquisition incentives flip with a positively skewed macro shock and frequent (as opposed to rare) crises. To see this, observe that with a positive skewness, $s \in (0,1)$, the negative macro shock is likely and with a sufficiently high Δ it is associated with weak fundamentals as before (i.e. $s = \frac{\mu}{\Delta} + \iota$, with $\iota > 0$ as in the Proof of Proposition 2). Different to our model setup crises in region 2 are now frequent and the Bayesian updating channel is stronger after not observing a crisis. Following arguments analog to the ones in our paper, we find that the value of information is higher after *not* observing a crisis, i.e. $v(0,0) > v(1,1)$, under the conditions of a modified Assumption 1 with $0 < s < \bar{s} < 1$ and $\Delta > \underline{\Delta} > 0$.

5.4 Alternative interpretation of exposure

One could consider an alternative model setup, where learning is not about the realization of the macro shock but about whether the two regions are exposed to the macro shock itself. In this setup, two macro shock realizations suffice, so $1-p-q=0$. That is, the macro shock realization is either positive or negative and ex-post both regions are either exposed to the macro shock or not, where the scenario of no exposure to the macro shock is equivalent to $m=0$ in our model.

As before, both regions are potentially exposed ex-ante via the common macro shock. Observing a crisis in region 1 and learning about an exposure to the macro shock ex-post suggest that the fundamentals in region 2 are likely to be affected by a negative macro shock that also contributed to the crisis in region 1. Conversely, learning about no exposure to the macro shock ex-post after observing a crisis in region 1 is favorable information for the local fundamentals in region 2. However, not observing a crisis in region 1 would still imply a more favorable view about the fundamental in region 2 due to the Bayesian updating channel, because it induces a positive view about the macro shock realization and the exposure to the macro shock has positive weight. Hence, by comparing the conditional crisis probabilities for the latter two cases, the wake-up call component of contagion can be isolated in the same way in this alternative model setup. Also the incentives to acquire information about the macro shock in such an alternative setup are similar to the present version, meaning that the key mechanism stays the same.

5.5 Other robustness issues

We discuss several robustness issues in this section. Our analytical results are derived under the conditions of Assumption 1. Given that the conditions might seem restrictive, it is worth noting that they are sufficient but not necessary for our results (see Figure 3, for instance). Most importantly, the benchmark parameter values used for the numerical analysis provided in the figures illustrate that wake-up call contagion also holds for a high probability of the zero macro shock, suggesting that the bound η is merely relevant for analytical tractability. Also the bounds on the precision of private and public signals are not more stringent than the standard sufficient conditions for equilibrium uniqueness in global games models (Morris and Shin 2003; Svensson 2006).

Next, our model setup abstracts from information acquisition in region 1 to simplify the exposition. This allows us to focus on how the wake-up call of a crisis in region 1 affects the incentives to acquire information in region 2 and may therefore result in contagion. Allowing for information acquisition in region 1 does not affect our main insights. For some intermediate region of information costs, there is a unique equilibrium with no information acquisition in region 1 and information acquisition in region 2 only after a crisis in region 1.

Below we discuss two additional extensions and an alternative modeling approach. First, an important channel of our paper is how a wake-up call affects the incentives of investors in region 2 to acquire information about the macro shock. An additional channel of interest could be private information acquisition with convex costs (Szkup and Trevino 2015), whereby investors improve the precision of their private information at a cost after the wake-up call. It can be shown that the effect of wake-up call contagion is even larger when private information acquisition is allowed.

Second, we have so far considered the case of a perfect signal about the macro shock. The advantage of a perfectly revealing signal is that we can cleanly isolate the wake-up call component of contagion. A generalization to noisy signals is possible without altering the key mechanisms. One approach is to assume that investors only observe the publicly available signal with probability $z \in (0,1)$ upon incurring the information acquisition cost. More concretely, the hiring of analysts only leads with a certain probability to a conclusive understanding of the institutional characteristics such as structural or policy distortions that are shared across regions. As a result, there is always a positive mass of investors who remain uninformed. This variation of our model is already captured by our analysis of the general case when $0 < n_2 < 1$. Another approach is to consider an environment where investors who incur the cost always observe a signal about the macro shock, but they do not know whether the signal is correct. In this case, we have to use the mixture distribution approach also for informed investors, which adds an additional layer of complexity. Again, in a modified setup it would not be possible to cleanly isolate the wake-up call component of contagion as in our main model in which all informed investors observe $m = 0$.

6 Conclusion

We offer a theory of financial contagion to explain how wake-up calls may transmit crises. Our theory builds on global coordination games of regime change that are often applied to currency attacks, bank runs, and debt crises. Different to the existing literature, our model of contagion features sequential global regime-change games in two regions that are linked by an initially unobserved macro shock. The potential link between regional fundamentals is commonly known (ex-ante exposure) but its actual ex-post relevance (ex-post exposure) is only learned upon a fundamental re-assessment in which investors acquire costly information about the macro shock.

A crisis in region 1 is a wake-up call for investors in region 2 and induces them to re-assess the local fundamental in region 2. Since crises are rare events and the macro shock is negatively skewed, investors have an incentive to acquire information only after this wake-up call. The crisis probability in region 2 is higher after a crisis in region 1 than after no crisis, even if investors learn that the macro shock is zero and, hence, that there is no ex-post exposure to the crisis in region 1. Our theoretical contribution is to isolate the wake-up call component of contagion.

We explore normative and testable implications of the model. Consistent with empirical evidence, the strength of the wake-up call contagion effect depends on the extent of ex-ante exposure to a ground zero crisis country or bank, which potentially shares common vulnerabilities such as institutional weaknesses in macroeconomic policies, the quality of governance and regulation, the credibility of government guarantees and the proneness to financial misconduct or potential interbank exposures. Moreover, we find that information acquisition incentives increase not only after a crisis elsewhere is observed, but also in the extent of ex-ante exposure. We discuss avenues for future empirical work based on the model's implications. In our normative analysis, we find that opacity can be good when information acquisition is associated with a higher ex-ante crisis probability. We also find that greater transparency can increase the incentives to acquire information.

References

Acemoglu, D. and A. Scott (1997). Asymmetric business cycles: Theory and time-series evidence. *Journal of Monetary Economics 40*, 501–533.

Acharya, V. V. and T. Yorulmazer (2008). Information contagion and bank herding. *Journal of Money, Credit and Banking 40*(1), 215–231.

Adrian, T., N. Boyarchenko, and D. Giannone (2019). Vulnerable Growth. *American Economic Review 109*(4), 1263–89.

Ahnert, T. and A. Kakhbod (2017). Information choice and amplification of financial crises. *Review of Financial Studies 30*(6), 2130–78.

Allen, F., A. Babus, and E. Carletti (2012). Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics 104*(3), 519–534.

Allen, F. and D. Gale (2000). Financial contagion. *Journal of Political Economy 108*(1), 1–33.

Ammer, J. and J. Mei (1996). Measuring International Economic Linkages with Stock Market Data. *Journal of Finance 51*, 1743–1764.

Armelius, H., C. Bertsch, I. Hull, and X. Zhang (2020). Spread the word: International spillovers from central bank communication. *Journal of International Money and Finance 103*.

Bae, J., C.-J. Kim, and C. R. Nelson (2007). Why are stock returns and volatility negatively correlated? *Journal of Empirical Finance 14*(1), 41 – 58.

Bannier, C. E. and F. Heinemann (2005). Optimal Transparency and Risk-Taking to Avoid Currency Crises. *Journal of Institutional and Theoretical Economics 161*, 374–391.

Barro, R. J. (2006). Rare Disasters and Asset Markets: Consumption Disasters in the Twentieth Century. *Quarterly Journal of Economics 121*(3), 823–866.

Basu, R. (1998). Contagion crises: the investors' logic. *Mimeo*.

Bonfim, D. and J. A. C. Santos (2020). The Importance of Deposit Insurance Credibility. *Banco de Portugal Working Papers*.

Buncic, D., J. Li, P. van Santen, P. Wallin, and J. Winstrand (2019). The Riksbank's method for stress testing banks' capital. *Sveriges Riksbank Staff Memo*.

Calvo, G. A. and E. G. Mendoza (2000). Rational contagion and the globalization of securities markets. *Journal of International Economics 51*, 79–113.

Campbell, J. Y. and L. Hentschel (1992). No news is good news. *Journal of Financial Economics 31*(3), 281 – 318.

Carlsson, H. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica 61*(5), 989–1018.

Chen, H., Y. K. Lu, and W. Suen (2016). The Power of Whispers: A Theory of Rumor, Communication and Revolution. *International Economic Review 57*(1).

Chen, Y. (1999). Banking panics: The role of the first-come, first-served rule and information externalities. *Journal of Political Economy 107*(5).

Colombo, L., G. Femminis, and A. Pavan (2014). Information Acquisition and Welfare. *Review of Economic Studies 81*, 1438–1483.

Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin (2004). Does One Soros Make a Difference? A Theory of Currency Crisis with Large and Small Traders. *Review of Economic Studies 71*(1), 87–113.

Corsetti, G., B. Guimaraes, and N. Roubini (2006). International lending of last resort and moral hazard: A model of IMF's catalytic finance. *Journal of Monetary Economics 53*(3), 441–471.

Corsetti, G., P. Pesenti, and N. Roubini (1999). What caused the Asian currency and financial crisis? *Japan and the world economy 11*(3), 305–373.

Da, Z., J. Engelberg, and P. Gao (2011). In Search of Attention. *Journal of Finance 66*(5), 1461–99.

Dang, T. V., G. B. Gorton, and B. Holmström (2015). Ignorance, Debt and Financial Crises. *Mimeo*.

Dasgupta, A. (2004). Financial Congtagion Through Capital Connections: A Model of the Origin and Spread of Bank Panics. *Journal of the European Economic Association 2*, 1049–1084.

Dasgupta, A., R. Leon-Gonzales, and A. Shortland (2011). Regionality revisited: An example of the direction of spread of currency crises. *Journal of International Money and Finance 30*, 831–848.

Drazen, A. (1999). Political Contagion in Currency Crises. *NBER Working Paper 7211*.

Dyer, T., M. Lang, and L. Stice-Lawrence (2017). The evolution of 10-k textual disclosure: Evidence from latent dirichlet allocation. *Journal of Accounting and Economics 64*(2-3), 221–245.

Edmond, C. (2013). Information Manipulation, Coordination, and Regime Change. *Review of Economic Studies 80*(4), 1422–1458.

Flood, R. and P. Garber (1984). Collapsing exchange-rate regimes: Some linear examples. *Journal of International Economics 17*(1-2), 1–13.

Forbes, K. J. (2012). The 'Big C': Identifying and Mitigating contagion. *The Changing Policy Landscape. 2012 Jackson Hole Symposium*, 23–87.

Giordano, R., M. Pericoli, and P. Tommasino (2013). Pure or Wake-up-Call Contagion? Another Look at the EMU Sovereign Debt Crisis. *International Finance 16*(2), 131–160.

Glick, R. and A. K. Rose (1999). Contagion and trade: Why are currency crises regional? *Journal of International Money and Finance 18*, 603–617.

Goldstein, I. and A. Pauzner (2004). Contagion of Self-Fulfilling Financial Crises Due to Diversification of Investment Portfolios. *Journal of Economic Theory 119*, 151–183.

Goldstein, I. and A. Pauzner (2005). Demand Deposit Contracts and The Probability of Bank Runs. *Journal of Finance 60*(3), 1293–1327.

Goldstein, M. (1998). The Asian financial crisis causes, cures, and systematic implications. *Washington D.C.: Institute for International Economics*.

Gorton, G. and G. Ordoñez (2014). Collateral Crises. *American Economic Review 104*(2), 343–378.

Gorton, G. and G. Ordoñez (2019). Good Booms, Bad Booms. *Journal of the European Economic Association (forthcoming)*.

Heinemann, F. and G. Illing (2002). Speculative attacks: unique equilibrium and transparency. *Journal of International Economics 58*, 429–450.

Hellwig, C. and L. Veldkamp (2009). Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies 76*, 223–251.

Johnson, S., P. Boone, A. Breach, and E. Friedman (2000). Corporate governance in the asian financial crisis. *Journal of Financial Economics 58*, 141–186.

Jovanovic, B. (2006). Asymmetric Cycles. *Review of Economic Studies 73*, 145–162.

Karas, A., W. Pyle, and K. Schoors (2013). Deposit Insurance, Banking Crises, and Market Discipline: Evidence from a Natural Experiment on Deposit Flows and Rates. *Journal of Money, Credit and Banking 45*, 179–200.

Kiyotaki, N. and J. Moore (2002). Balance-sheet contagion. *The American Economic Review 92*(2), 46–50.

Kodres, L. E. and M. Pritsker (2002). A Rational Expectations Model of Financial Contagion. *Journal of Finance 57*(2), 769–799.

Krugman, P. (1979). A model of balance-of-payments crises. *Journal of Money, Credit and Banking 11*(3), 311–25.

Manz, M. (2010). Information-based contagion and the implications for financial fragility. *European Economic Review 54*, 900–910.

Milgrom, P. R. (1981). Good News and Bad News: Representation Theorems and Applications. *Bell Journal of Economics 12*(2), 380–91.

Mitton, T. (2002). A cross-firm analysis of the impact of corporate governance on the east asian financial crisis. *Journal of Financial Economics 64*, 215–241.

Mondria, J. and C. Quintana-Domeque (2013). Financial Contagion and Attention Allocation. *Economic Journal 123*(May), 429–54.

Morris, S. and H. S. Shin (1998). Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review 88*(3), 587–597.

Morris, S. and H. S. Shin (2002). Social Value of Public Information. *American Economic Review 92*(5), 1521–1534.

Morris, S. and H. S. Shin (2003). Global games: theory and applications. In M. Dewatripont, L. P. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)*. Cambridge University Press.

Morris, S. and H. S. Shin (2004). Coordination risk and the price of debt. *European Economic Review 48*(1), 133–153.

Neftçi, S. N. (1984). Are economic time series asymmetric over the business cycle? *Journal of Political Economy 92*(2).

Obstfeld, M. (1986). Rational and Self-fulfilling Balance-of-Payments Crises. *American Economic Review 76*(1), 72–81.

Oh, F. D. (2013). Contagion of a liquidity crisis between two firms. *Journal of Financial Economics 107*, 386–400.

Pavlova, A. and R. Rigobon (2008). The Role of Portfolio Constraints in the International Propagation of Shocks. *Review of Economic Studies 75*(4), 1215–56.

Rancière, R., A. Tornell, and F. Westermann (2003). Crises and Growth: A Re-evaluation. *NBER Working Paper 10073*.

Rancière, R., A. Tornell, and F. Westermann (2008). Systemic Crises and Growth. *Quarterly Journal of Economics 123*(1), 359–406.

Rochet, J.-C. and X. Vives (2004). Coordination failures and the lender of last resort: was bagehot right after all? *Journal of the European Economic Association 2*(6), 1116–1147.

Shadmehr, M. and D. Bernhardt (2015). State censorship. *American Economic Journal: Microeconomics 7*(2), 280–307.

Svensson, L. E. O. (2006). Social Value of Public Information: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con. *American Economic Review 96*(1), 448–452.

Szkup, M. and I. Trevino (2015). Information acquisition and transparency in global games. *Journal of Economic Theory 160*, 387–428.

Taketa, K. (2004). Contagion of Currency Crises across Unrelated Countries. *IMES Discussion Paper No. 2004-E-22*.

Van Rijckeghem, C. and B. Weder (2001). Sources of contagion: is it finance or trade? *Journal of International Economics 54*(2), 293–308.

Van Rijckeghem, C. and B. Weder (2003). Spillovers through banking centers: a panel data analysis of bank flows. *Journal of International Money and Finance 22*(4), 483–509.

Veldkamp, L. (2005). Slow Boom, Sudden Crash. *Journal of Economic Theory 124*, 230–257.

Vives, X. (2005). Complementarities and Games: New Developments. *Journal of Economic Literature 43*, 437–479.

Vlastakis, N. and R. N. Markellos (2012). Information demand and stock market volatility. *Journal of Banking & Finance 36*(6), 1808–1821.

A Online Appendix

A.1 Equilibrium in region 1

To simplify the exposition, we focus on the case of uninformed investors, $n_1 = 0$. We first discuss Bayesian updating of uninformed investors receiving a private signal x_{i1} about Θ_1 and derive the equilibrium conditions in Section A.1.1. Next, we prove Lemma 1 in Section A.1.2.

A.1.1 Deriving the equilibrium in region 1 for the case $n_1 = 0$

Bayesian updating. Uninformed investors in region 1 use Bayes' rule to form a belief about the macro shock, where $\hat{p} \equiv \Pr\{m = \Delta | x_{i1}\}$, and $\hat{q} \equiv \Pr\{m = -s\Delta | x_{i1}\}$:

$$\hat{p} = p \Pr\{x_{i1} | m = \Delta\} \Gamma_1^{-1} \qquad \hat{q} = q \Pr\{x_{i1} | m = -s\Delta\} \Gamma_1^{-1}, \tag{12}$$

where $\Gamma_1 = p \Pr\{x_{i1}|m = \Delta\} + q \Pr\{x_{i1}|m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1}|m = 0\}$ and:

$$\Pr\{x_{i1}|m\} = \frac{1}{\sqrt{\mathrm{Var}[x_{i1}|m]}} \phi\left(\frac{x_{i1} - \mathbb{E}[x_{i1}|m]}{\sqrt{\mathrm{Var}[x_{i1}|m]}}\right) = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i1} - (\mu + m)}{\sqrt{\frac{1}{\alpha} + \frac{1}{\beta}}}\right).$$

Using $p = qs$, we obtain $\frac{d\hat{p}}{dx_{i1}} > 0$, $\frac{d\hat{q}}{dx_{i1}} < 0$, and $\frac{d(1-\hat{p}-\hat{q})}{dx_{i1}} \left[x_{i1} - \mu + \frac{1-s}{2}\Delta\right] \leq 0$, with strict inequality if $x_{i1} \neq \mu + \frac{1-s}{2}\Delta$. An investor places more weight on the probability of a positive (negative) macro shock after a higher (lower) private signal. The relationship between the posterior probability of a zero macro shock and the private signal, x_{i1}, is non-monotone. It increases if $x_{i1} > \underline{x_1}(s, \Delta) \equiv \mu + \frac{1-s}{2}\Delta$. The bound is below μ if the macro shock is negatively skewed ($s > 1$).

Equilibrium conditions. For the case of $n_1 = 0$, the system of equations comprises the critical mass and indifference condition for region 1. The critical mass condition states that the proportion of attacking investors $A_1^*(m)$ equals the fundamental threshold $\Theta_1^*(m)$ for each realized m:

$$\Theta_1^*(m) = \Phi\left(\sqrt{\beta}[x_1^* - \Theta_1^*(m)]\right), \forall m \in \{-s\Delta, 0, \Delta\}. \tag{13}$$

Given the invariant attack rule, the fundamental thresholds are equal, $\Theta_1 \equiv \Theta_1^*(m)$, $\forall m$. The indifference condition states that an uninformed investor with threshold signal $x_{i1} = x_1^*$ is indifferent whether to attack:

$$\hat{p}^*\Psi(\Theta_1^*, x_1^*, \Delta) + \hat{q}^*\Psi(\Theta_1^*, x_1^*, -s\Delta) + (1 - \hat{p}^* - \hat{q}^*)\Psi(\Theta_1^*, x_1^*, 0) \equiv J(\Theta_1^*, x_1^*) = \gamma_1, \tag{14}$$

where $\hat{p}^* = \hat{p}(x_1^*)$, $\hat{q}^* = \hat{q}(x_1^*)$ and $\Psi(\Theta_1^*, x_1^*, m) \equiv \Phi\big(\Theta_1^*\sqrt{\alpha+\beta} - \frac{\alpha(\mu+m)+\beta x_1^*}{\sqrt{\alpha+\beta}}\big)$. Solving equation (13) for x_1^* and plugging into equation (14), we arrive at one equation in one unknown.

Monotone equilibria. Using the results of Milgrom (1981) and Vives (2005), we can show that the best-response function of an individual investor strictly increases in the threshold used by other investors. Using Proposition 1 of Milgrom (1981), we conclude that $\Pr\{\Theta_1 \leq \Theta_1^* | x_{i1}\}$ monotonically decreases in x_{i1}. Hence, $\frac{d\Pr\{\Theta_1 \leq \Theta_1^* | \hat{x}_1\}}{d\Theta_1^*} > 0$. Equation (14) then implies:

$$0 \leq \frac{d\hat{\Theta}_1(\hat{x}_1)}{d\hat{x}_1} \leq \left(1 + \sqrt{2\pi\beta^{-1}}\right)^{-1}. \tag{15}$$

Thus, our focus on monotone equilibria is valid. Equation (15) is used to determine conditions sufficient for a unique monotone Bayesian equilibrium in Lemma 1.

A.1.2 Proof of Lemma 1

The proof consists of two steps. First, we show that $J(\Theta_1, x_1) \equiv J(\Theta_1) \to 1 > \gamma_1$ as $\Theta_1 \to 0$, and $J(\Theta_1) \to 0 < \gamma_1$ as $\Theta_1 \to 1$. Second, we show that $\frac{dJ(\Theta_1)}{d\Theta_1} < 0$ for some sufficiently high but finite values of β, such that J strictly decreases in Θ_1. We denote this lower bound as $\underline{\beta}_1$. Therefore, if Θ_1^* exists, it is unique. Notably, this argument implicitly defines the lower and upper dominance regions of the game. However, as Θ_1 can be any real number, the limit used here is one-sided.

Step 1 (limiting behavior): We solve equation (13) for x_1^*, plug into equation (14) and let $\Psi(\Theta_1, x_1, m) \equiv \Psi(\Theta_1, m)$. Observe that $J(\Theta_1)$ is a weighted average of the $\Psi(\Theta_1, m)$'s evaluated at the different levels of m. As $\Theta_1 \to 0$, then $\Psi(\Theta_1, m) \to 1$ for any $m \in \{-s\Delta, 0, \Delta\}$, so $J(\Theta_1) \to 1 > \gamma_1$. Likewise, as $\Theta_1 \to 1$, then $\Psi(\Theta_1, m) \to 0$ for any $m \in \{-s\Delta, 0, \Delta\}$, so $J(\Theta_1) \to 0 < \gamma_1$.

Step 2 (strictly negative slope): The total derivative of J is:

$$\begin{aligned}\frac{dJ(\Theta_1)}{d\Theta_1} &= \hat{p}(x_1(\Theta_1))\frac{d\Psi(\Theta_1,\Delta)}{d\Theta_1}+\hat{q}(x_1(\Theta_1))\frac{d\Psi(\Theta_1,-s\Delta)}{d\Theta_1}\\ &+ (1-\hat{p}(x_1(\Theta_1))-\hat{q}(x_1(\Theta_1)))\frac{d\Psi(\Theta_1,0)}{d\Theta_1}\\ &+ \frac{d\hat{p}(x_1(\Theta_1))}{dx_1}\frac{dx_1(\Theta_1)}{d\Theta_1}\left[\Psi(\Theta_1,\Delta)-\Psi(\Theta_1,0)\right]\\ &+ \frac{d\hat{q}(x_1(\Theta_1))}{dx_1}\frac{dx_1(\Theta_1)}{d\Theta_1}\left[\Psi(\Theta_1,-s\Delta)-\Psi(\Theta_1,0)\right]. \end{aligned} \tag{16}$$

The proof proceeds by inspecting the individual terms of equation (16). For the analysis of the special case where all investors are informed, $n_1 = 1$, we can use a result from standard global games models: $\frac{d\Psi(\Theta_1,m)}{d\Theta_1} < 0$ if $\beta > \frac{\alpha^2}{2\pi}$ for all m. Thus, the first three components of the sum are negative and finite for sufficiently high but finite private noise. The sign of the two terms in square brackets in the last two summands in (16) is negative and positive, respectively: $\Psi(\Theta_1^*,\Delta) \leq \Psi(\Theta_1^*,0)$ and $\Psi(\Theta_1^*,\Delta) \geq \Psi(\Theta_1^*,0)$. However, the difference vanishes in the limit when $\beta \to \infty$. The last terms to consider are $\frac{d\hat{p}(x_1(\Theta_1))}{dx_1(\Theta_1)}\frac{dx_1}{d\Theta_1}$ and $\frac{d\hat{q}(x_1(\Theta_1))}{dx_1(\Theta_1)}\frac{dx_1}{d\Theta_1}$. Given the previous sufficient conditions on the relative precision of the private signal:

$$0 < \frac{dx_1}{d\Theta_1} = 1 + \left(\sqrt{\beta}\phi(\Phi^{-1}(\Theta_1))\right) < 1 + \sqrt{2\pi\alpha^{-1}}.$$

The derivative is finite for $\beta \to \infty$. Taken together with the zero limit of the first factor of the third and forth term, this terms vanish in the limit. Note that for $\beta \to \infty$ the updated prior distribution becomes degenerate. We have $\hat{p} = 1$ for $x > \mu + \frac{\Delta}{2}$ and $\hat{p} = 0$ for $x < \mu + \frac{\Delta}{2}$. Moreover, $1 - \hat{p} - \hat{q} = 1$ for $\mu + \frac{-s\Delta}{2} < x < \mu + \frac{\Delta}{2}$ and $\hat{q} = 1$ for $x < \mu + \frac{-s\Delta}{2}$. Clearly, there are some discontinuities. At the same time, it must be that any Θ_1^* and $x_1^* \approx \Theta_1^*$ solving the system has to be very close to μ for large values of β. As a result, it is guaranteed that $\frac{d\hat{p}(x_1(\Theta_1))}{dx_1} = 0$ and $\frac{d\hat{q}(x_1(\Theta_1))}{dx_1} = 0$ is in the permissible range. Hence, by continuity, there exists a finite precision level $\beta > \underline{\beta}_1 \in (0,\infty)$ such that $\frac{dJ(\Theta_1)}{d\Theta_1} < 0$ for all $\beta > \underline{\beta}_1$. This concludes the proof of Lemma 1.

A.2 Equilibrium in region 2

To study the equilibrium in region 2, we first analyze the coordination stage in Section A.2.1. The main results are on Bayesian updating and on the existence of unique attack rules are summarized in Lemma 2 and Corollary 4, respectively. Next, we analyze the information stage in Sections A.2.2 and A.2.3. The main results are summarized in Lemma 3, which describes how fundamental and signal thresholds depend on the proportion of informed investors and in Lemma 4, which establishes a strategic complementarity in information choices. Finally, we prove Proposition 2 in Section A.2.4 and Proposition 1 in Section A.2.5.

A.2.1 Coordination stage in region 2

The optimal behavior of investors in region 2 at the coordination stage can be described by extending the results from region 1. Investors use the information about region 1 to update their prior about their beliefs about the distribution of the macro shock, using Bayes' rule:

$$p' \equiv \Pr\{m = \Delta | f\} = p \Pr\{f | m = \Delta\} \, \Gamma_2^{-1} \tag{17}$$

$$q' \equiv \Pr\{m = -s\Delta | f\} = q \Pr\{f | m = -s\Delta\} \, \Gamma_2^{-1}, \tag{18}$$

with $\Pr\{f = 1|m\} = \Pr\{\Theta_1 < \Theta_1^*|m\}$ and $\Gamma_2 \equiv p\Pr\{f|m = \Delta\} + q\Pr\{f|m = -s\Delta\} + (1 - p - q)\Pr\{f|m = 0\}$, where $f = 1$ corresponds to a crisis and $f = 0$ corresponds to no crisis.

Lemma 2 states the evolution of the beliefs about the macro shock.

Lemma 2 ***Beliefs about the macro shock.*** *The wake-up call of a crisis in region* 1 *is associated with less favorable beliefs about the macro shock, while no crisis in region* 1 *is associated with more favorable beliefs about the macro shock:*

$$\begin{cases} p' < p, \; q' > q & if\ f = 1 \\ p' > p, \; q' < q & if\ f = 0. \end{cases}$$

Moreover, we can state that:

$$\begin{cases} \frac{p'}{1-q'} < \frac{p}{1-q}, \frac{q'}{1-p'} > \frac{q}{1-p} & if\ f = 1\ and\ n_1 \in \{0,1\} \\ \frac{p'}{1-q'} > \frac{p}{1-q}, \frac{q'}{1-p'} < \frac{q}{1-p} & if\ f = 0\ and\ n_1 \in \{0,1\}. \end{cases}$$

The first set of inequalities are an extension of a comparative static in Morris and Shin (2003) and Vives (2005). For the special case of $n_1 = 1$, we have $\frac{d\Theta_1^*(1,m)}{dm} < 0$. Similarly for the general case, a more favorable information about fundamentals is associated with a lower fundamental threshold. The results follow from Bayesian updating in equations (17) and (18). The second set of inequalities on the right-hand side follow from $\frac{d}{dm}\big(\Pr\{f=1|m\}-\Pr\{f=0|m\}\big)<0$. The results are immediate for $n_1\in\{0,1\}$ and also hold for the general case, $n_1\in[0,1]$, if the thresholds are monotone in n_t. We show this monotonicity in Lemma 3.

Using the updated p' and q' as weights, the belief about Θ_2 prior to receiving a private signal x_{i2} follows again a mixture distribution. It is an average over the cases of negative, zero and positive macro shocks with weights depending on f:

$$\Theta_2|f \equiv p'\left[\Theta_2|m=-s\Delta\right]+q'\left[\Theta_2|m=\Delta\right]+(1-p'-q')\left[\Theta_2|m=0\right]. \tag{19}$$

For the general case of $n_2\in[0,1]$ we have seven equations in seven unknowns. Three critical mass conditions state that the proportion of attacking investors $A_2^*(m)$ equals the fundamental threshold $\Theta_2^*(m)$ for each realized $m\in\{-s\Delta,0,\Delta\}$:

$$\Theta_2^*(m)=n_2\Phi\big(\sqrt{\beta}[x_I^*(m)-\Theta_2^*(m)]\big)+(1-n_2)\Phi\big(\sqrt{\beta}[x_U^*-\Theta_2^*(m)]\big), \tag{20}$$

where the short-hands are $\Theta_2^*(m)\equiv\Theta_2^*(n_2,m)$, $x_I^*(m)\equiv x_{2I}^*(n_2,m)$, and $x_U^*\equiv x_{2U}^*(n_2)$ for the fundamental threshold and the signal thresholds of informed and uninformed investors, respectively.

The first indifference condition states for each $n_2\in[0,1]$ that an uninformed investor with threshold signal $x_{i2}=x_U^*$ is indifferent whether to attack:

$$\begin{aligned} & J(n_2,\Theta_2^*(\Delta),\Theta_2^*(-s\Delta),\Theta_2^*(0),x_U^*) \\ \equiv\ & \hat{p}'^*\Psi(\Theta_2^*(\Delta),x_U^*,\Delta)+\hat{q}'^*\Psi(\Theta_2^*(-s\Delta),x_U^*,-s\Delta)+(1-\hat{p}'^*-\hat{q}'^*)\Psi(\Theta_2^*(0),x_U^*,0)=\gamma_2 \end{aligned} \tag{21}$$

where $\hat{p}'^*=\hat{p}'(x_U^*)$ and $\hat{q}'^*=\hat{q}'(x_U^*)$ solve equation (12) after replacing p and q with p' and q'. Moreover, $\Psi(\Theta_2^*(m),x_d^*,m)\equiv\Phi\big(\Theta_2^*\sqrt{\alpha+\beta}-\frac{\alpha(\mu+m)+\beta x_d^*}{\sqrt{\alpha+\beta}}\big)$ for $d\in\{I,U\}$ and $m\in\{-s\Delta,0,\Delta\}$.

Three additional indifference conditions, one for each realized macro shock, state that an informed investor is indifferent between attacking or not upon receiving the signal $x_{i2}=x_I^*(m)$:

$$\Psi\left(\Theta_2^*(n_2,m),x_I^*(m),m\right)=\gamma_2\ \ \forall\, m\in\{-s\Delta,0,\Delta\}. \tag{22}$$

For the special case of the equilibrium in region 1 with $n_1=0$, we had two thresholds x_1^* and Θ_1^* for each m. There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states x_1^* in terms of Θ_1^*, into the indifference condition. This yields one equation implicit in Θ_1^*. We pursue a similar strategy here and express the equilibrium in terms of $\Theta_2^*(-s\Delta)$, $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$ only.

To simplify the system of equations, we can use the following insight. Since uninformed investors do not observe the macro shock realization, the signal threshold must be identical across these realizations, $x_U^* \equiv x_U^*(-s\Delta) = x_U^*(0) = x_U^*(\Delta)$. In the following steps, we derive this threshold for either realization of m by using $\Theta_2^*(m)$ and equalize both expressions. First, we use the critical mass conditions in equation (20) for $\Theta_2^*(m)$ to express x_U^* as a function of each $\Theta_2^*(m)$ and $x_I^*(m)$. Second, we use the indifference condition of informed investors for each m to obtain $x_I^*(m)$ as a function of $\Theta_2^*(m)$. Thus, $\forall m$:

$$x_U^*(m) = \Theta_2^*(m) + \Phi^{-1}\left(\frac{\Theta_2^*(m) - n_2\Phi\left(\frac{\alpha(\Theta_2^*(m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}\right)/\sqrt{\beta}. \tag{23}$$

Hence, for $m \in \{-s\Delta, 0, \Delta\}$, there exists a $\underline{\beta}_2 \in (0,\infty)$ such that for all $\beta > \underline{\beta}_2$: $\frac{dx_U^*(m)}{d\Theta_2^*(m)} > 0$.

Since the signal threshold is the same for an uninformed investor, subtracting equation (23) evaluated at $m = 0$ from the same equation evaluated at $m = -s\Delta$ or at $m = \Delta$ must yield zero. This yields the first two pair-wise implicit relationships between $\Theta_2^*(-s\Delta)$, $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$:

$$K_1(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0)) \equiv x_U^*(0) - x_U^*(-s\Delta) = 0 \tag{24}$$

$$K_2(n_2, \Theta_2^*(0), \Theta_2^*(\Delta)) \equiv x_U^*(0) - x_U^*(\Delta) = 0. \tag{25}$$

Now, we construct the third implicit relationship between the three aggregate thresholds by inserting equation (23) evaluated at each m in $\Psi(\Theta_2^*(m), x_U^*(m), m)$, respectively, and in $\hat{p}(p')$ and $\hat{q}(q')$ as used in J:

$$L(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) \equiv J(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) = \gamma_2. \tag{26}$$

Corollary 4 establishes existence and uniqueness for a given $n_2 \in [0,1]$ under the conditions of Assumption 1 by analyzing the system of equations given by (24), (25) and (26).

Corollary 4 ***Existence of unique attack rules in region 2.*** *If private information is sufficiently precise, then for any proportion of informed investors in region* 2, $n_2 \in [0,1]$, *there exist unique* *attack rules for informed investors, $a_I^*(m,\cdot)$, and for uniformed investors, $a_U^*(\cdot)$.*

Proof The first and second equation depend only on two thresholds, $K_1(n_2,\Theta_2^*(-s\Delta),\Theta_2^*(0))=0$ and $K_2(n_2,\Theta_2^*(0),\Theta_2^*(\Delta))=0$, while the third depends on all three, $L(n_2,\Theta_2^*(-s\Delta),\Theta_2^*(0),\Theta_2^*(\Delta))=\gamma_2$. In a first step, we analyze, for a given n_2, the relationship between $\Theta_2(-s\Delta)$ and $\Theta_2(0)$, as governed by K_1. We obtain $\frac{\partial K_1}{\partial\Theta_2^*(0)}>0$, $\frac{\partial K_1}{\partial\Theta_2(-s\Delta)}<0$, and $\frac{\partial K_1}{\partial\Theta_2(\Delta)}=0$. Hence, $\frac{d\Theta_2(0)}{d\Theta_2(-s\Delta)}>0$ by the implicit function theorem. Likewise, we analyze the relationship between $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$, as governed by K_2. We obtain $\frac{\partial K_2}{\partial\Theta_2^*(0)}>0$, $\frac{\partial K_2}{\partial\Theta_2(-s\Delta)}=0$, and $\frac{\partial K_2}{\partial\Theta_2(\Delta)}<0$. Hence, $\frac{d\Theta_2(0)}{d\Theta_2(\Delta)}>0$. These results do not require a bound on the precision of private information.

In a second step, we analyze, for a given n_2, the relationship between all three fundamental thresholds, as governed by L. We know from our analysis of the case of informed investors that $\frac{d\Psi(\Theta_2,m)}{d\Theta_2}<0$ for all m if $\beta>\frac{\alpha^2}{2\pi}$. Analogous to the argument in the proof of Lemma 1, there exists a sufficiently high but finite value of the private precision such that $\frac{\partial L}{\partial\Theta_2(m)}<0$ for all m. Hence, in the limit $\frac{d\Theta_2(0)}{d\Theta_2(-s\Delta)}<0$ for a given $\Theta_2(\Delta)$, $\frac{d\Theta_2(0)}{d\Theta_2(\Delta)}<0$ for a given $\Theta_2(-s\Delta)$, and $\frac{d\Theta_2(\Delta)}{d\Theta_2(-s\Delta)}<0$ for a given $\Theta_2(0)$. By continuity, there exists a finite precision of private information, $\underline{\beta}_2\in(0,\infty)$, that guarantees the inequality if $\beta>\underline{\beta}_2$.

In a third step, we establish uniqueness conditional on existence. Thus suppose for now that an equilibrium exists. Then, due to the monotonicity and the opposite signs of the respective derivatives, we have that there is a single crossing of K_1 and L in the $(\Theta_2(-s\Delta),\Theta_2(0))$ space and a single crossing of K_2 and L in the $(\Theta_2(\Delta),\Theta_2(0))$ space, as shown in Figure 4. Observe that this is a "partial equilibrium" argument since the third threshold is taken as given. We now move to a "general equilibrium" argument. Building on a second feature of the system, the opposite signs of the respective derivatives are not only a sufficient condition for single crossings in the two panels of Figure 4, but they also imply that $\Theta_2(-s\Delta)$ and $\Theta_2(0)$ are each decreasing in $\Theta_2(\Delta)$ (left panel), where an increase in $\Theta_2(\Delta)$ shifts the L curve inwards. Likewise, $\Theta_2(\Delta)$ and $\Theta_2(0)$ are each decreasing in $\Theta_2(-s\Delta)$ (right panel). Hence, starting from a general equilibrium, any modification of $\Theta_2(\Delta)$ and $\Theta_2(-s\Delta)$ must lead to a violation of the system of equations. Given $\frac{\partial L}{\partial\Theta_2(\Delta)}<0$ and $\frac{\partial L}{\partial\Theta_2(-s\Delta)}<0$, the combination of fundamental thresholds $(\Theta_2^*(-s\Delta),\Theta_2^*(0),\Theta_2^*(\Delta))$ that satisfies K_1 and L in the $(\Theta_2(-s\Delta),\Theta_2(0))$ space and K_2 and L in the $(\Theta_2(\Delta),\Theta_2(0))$ space is unique.

In a fourth step, we establish the existence of a combination of fundamental thresholds. Existence can be shown by proving the following sequence of points: (i) for the highest permissible

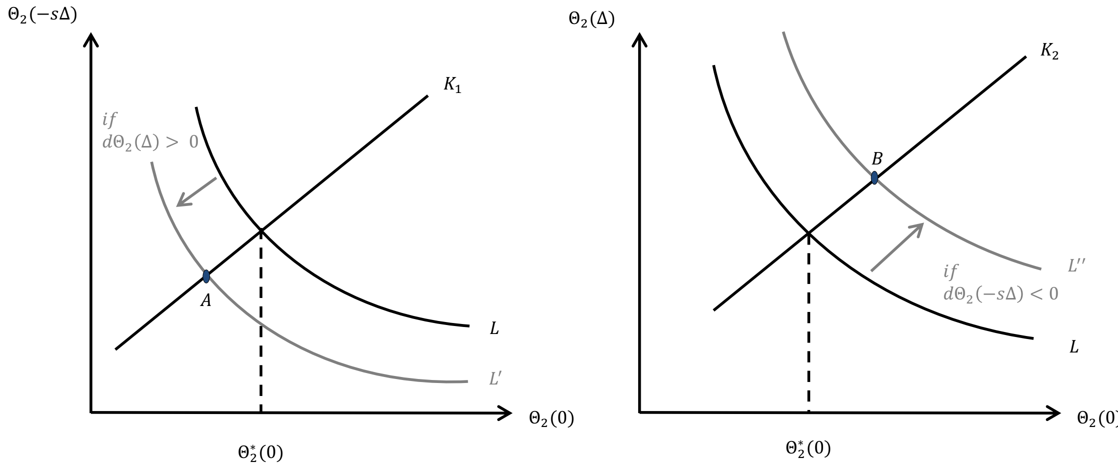

Figure 4: Single crossing.

value of $\Theta_2(-s\Delta)$, the value of $\Theta_2(0)$ prescribed by K_1 is strictly larger than the value of $\Theta_2(0)$ prescribed by L; (ii) for the lowest permissible value of $\Theta_2(-s\Delta)$, the value of $\Theta_2(0)$ prescribed by K_1 is strictly smaller than the value of $\Theta_2(0)$ prescribed by L; (iii) for the highest permissible value of $\Theta_2(\Delta)$, the value of $\Theta_2(0)$ prescribed by K_2 is strictly larger than the value of $\Theta_2(0)$ prescribed by L; (iv) for the lowest permissible value of $\Theta_2(\Delta)$, the value of $\Theta_2(0)$ prescribed by K_2 is strictly smaller than the value of $\Theta_2(0)$ prescribed by L; (v) for the lowest (highest) permissible value of $\Theta_2(-s\Delta)$, also $\Theta_2(0)$ must be at its lowest (highest) permissible value from K_1 and, hence, also $\Theta_2(\Delta)$ must be at its lowest (highest) permissible value from K_2, leading to a violation of L in both the $(\Theta_2(-s\Delta), \Theta_2(0))$ space and the $(\Theta_2(\Delta), \Theta_2(0))$ space; (vi) a successive increase (decrease) in $\Theta_2(0)$ shifts L continuously inwards (outwards) in both spaces until a fixed point is reached.

Before addressing points (i)-(iv), we start by analyzing the following auxiliary step. For any $\Theta_2(m) \geq \Theta_2^*(1,m)$, it can be shown that:

$$\frac{\partial}{\partial n_2}\Phi^{-1}\left(\frac{\Theta_2(m) - n_2\Phi\left(\frac{\alpha(\Theta_2(m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}\right) \geq 0 \tag{27}$$

because $J(1,\Theta_2) \leq \gamma_2$ for any m. Note that both the previous expression and the partial derivative hold with strict inequality if $\Theta_2(m) > \Theta_2^*(1,m)$. Inspecting the inside of the inverse of the cdf, Φ^{-1}, we define the highest permissible values of $\Theta_2(m)$ that are labeled $\overline{\Theta}_2(n_2,m)$ for all m:

$$1 = \frac{\overline{\Theta}_2(n_2,m) - n_2\Phi\left(\frac{\alpha(\overline{\Theta}_2(n_2,m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}. \tag{28}$$

Hence, $1 \geq \overline{\Theta}_2(1,m) \geq \Theta_2^*(1,m)\ \forall m$, where the first (second) inequality binds iff $n_2 = 0$ ($n_2 = 1$).

We now prove points (i) and (iii). Evaluate K_1 and K_2 at the highest permissible value, $\Theta_2(0) = \overline{\Theta}_2(n_2,0)$, which yields $\overline{\Theta}_2(n_2,-s\Delta)$ and $\overline{\Theta}_2(n_2,\Delta)$, respectively. Likewise, evaluate L at the highest permissible values, $\overline{\Theta}_2(n_2,0)$ and $\overline{\Theta}_2(n_2,-s\Delta)$, which yields $\Theta_2(\Delta) < \overline{\Theta}_2(n_2,\Delta)$. Similarly, evaluate L at $\overline{\Theta}_2(n_2,0)$ and $\overline{\Theta}_2(n_2,\Delta)$, which yields $\Theta_2(-s\Delta) < \overline{\Theta}_2(n_2,-s\Delta)$. This proves points (i) and (iii). Next, we proceed with points (ii) and (iv). We can similarly define the lowest permissible value of $\Theta_2(m)$, which is labeled $\underline{\Theta}_2(n_2,m)$ for all m. Now, $0 \leq \underline{\Theta}_2(1,m) \leq \Theta_2^*(1,m)\ \forall m$, where the first (second) inequality binds if and only if $n_2 = 0$ ($n_2 = 1$). Evaluate K_1 and K_2 at the lowest permissible value, $\Theta_2(0) = \underline{\Theta}_2(n_2,0)$, which yields $\underline{\Theta}_2(n_2,-s\Delta)$ and $\underline{\Theta}_2(n_2,\Delta)$, respectively. Likewise, evaluate L at the lowest permissible values, $\underline{\Theta}_2(n_2,0)$ and $\underline{\Theta}_2(n_2,-s\Delta)$, which yields $\Theta_2(\Delta) > \underline{\Theta}_2(n_2,\Delta)$. Similarly, evaluate L at $\underline{\Theta}_2(n_2,0)$ and $\underline{\Theta}_2(n_2,\Delta)$, which yields $\Theta_2(-s\Delta) > \underline{\Theta}_2(n_2,-s\Delta)$. This proves points (ii) and (iv). The proof of points (v)-(vi) follows, which completes the overall proof of Corollary 4.

A.2.2 Information stage in region 2: proportion of informed investors and thresholds

To characterize the value of information about the macro shock to investors in Appendix A.2.3, we first describe how the equilibrium fundamental and signal thresholds depend on the proportion of informed investors, as summarized below.

Lemma 3 ***Proportion of informed investors and equilibrium thresholds.*** *If Assumption 1 holds, then:*

(A) Boundedness. *The fundamental thresholds in case of informed investors bound the fundamental thresholds in case of asymmetrically informed investors:*

$$\Theta_2^*(1,\Delta) \leq \Theta_2^*(n_2,m) \leq \Theta_2^*(1,-s\Delta)\ \ \forall m \in \{-s\Delta, 0, \Delta\} \quad \forall n_2 \in [0,1]. \tag{29}$$

(B) Monotonicity in fundamental thresholds. *The fundamental threshold in the case of a negative (positive) macro shock increases (decreases) in the proportion of informed investors. Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded,* *that is* $\forall n_2 \in [0,1)$*:*

$$\frac{d\Theta_2^*(n_2,-s\Delta)}{dn_2} = \begin{cases} >0 \;\; if\; \Theta_2^*(n_2,-s\Delta) < \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) > \Theta_2^*(1,\Delta) \\ =0 \;\; if\; \Theta_2^*(n_2,-s\Delta) = \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) = \Theta_2^*(1,\Delta), \end{cases} \tag{30}$$

$$\frac{d\Theta_t^*(n_2,\Delta)}{dn_2} = \begin{cases} <0 \;\; if\; \Theta_2^*(n_2,-s\Delta) < \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) > \Theta_2^*(1,\Delta) \\ =0 \;\; if\; \Theta_2^*(n_2,-s\Delta) = \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) = \Theta_2^*(1,\Delta). \end{cases} \tag{31}$$

(C) Monotonicity in signal thresholds. *As a consequence of the monotonicity in fundamental thresholds:*

$$\frac{d(x_I^*(n_2,-s\Delta) - x_I^*(n_2,\Delta)))}{dn_2} \geq 0, \;\; \forall\, n_2 \in [0,1), \tag{32}$$

where $x_I^*(n_2,-s\Delta) - x_I^*(n_2,\Delta)) > 0, \;\; \forall\, n_2 \in [0,1]$.

(D) Uninformed investors. *If* $n_2 = 0$ *then:*

$$[x_I^*(n_2,m)|f=1] > [x_I^*(n_2,m)|f=0], \;\; \forall\, m \in \{\Delta,-s\Delta,0\}. \tag{33}$$

Proof We prove the results of Lemma 3 in turn. Since the argument applies for both regions, we use the subscript t. A general observation is that the updated belief on the probability of a positive macro shock becomes degenerate: $\hat{p} \to p$ for $\alpha \to 0$. Results (A) and (B) are closely linked, so we start with them. It will be useful to consider a modified system of equations where either K_1 or K_2 are used alongside $K_3(n_t,\Theta_t^*(-s\Delta),\Theta_t^*(\Delta)) \equiv x_{tU}^*(-s\Delta) - x_{tU}^*(\Delta) = 0$.

Results (A) and (B). This proof has three steps.

Step 1: We show in the first step that for $1-p-q \to 0$ the fundamental thresholds $\Theta_t^*(-s\Delta)$ and $\Theta_t^*(\Delta)$ in the case of asymmetrically informed investors lie either both within these bounds or outside of them. As a consequence of $\hat{p} \to p$, condition $L(n_t,\Theta_t^*(-s\Delta),\Theta_t^*(\Delta)) = 0$ prescribes that, for any n_t, the thresholds $\Theta_t^*(\Delta)$ and $\Theta_t^*(-s\Delta)$ are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all investors are informed, $\Theta_t^*(1,\Delta)$ and $\Theta_t^*(1,-s\Delta)$. This is proven by contradiction. First, suppose that $\Theta_t^*(\Delta) < \Theta_t^*(1,\Delta)$ and $\Theta_t^*(-s\Delta) < \Theta_t^*(1,-s\Delta)$. This leads to a violation of $L(\cdot) = 0$ because $J(\cdot) > \gamma_t \,\forall\, n_t$ if $\alpha \to 0$. Second, suppose that $\Theta_t^*(\Delta) > \Theta_t^*(1,\Delta)$ and $\Theta_t^*(-s\Delta) > \Theta_t^*(1,-s\Delta)$. Again, leading to a violation because $J(\cdot) < \gamma_t \,\forall\, n_t$ if $\alpha \to 0$. By continuity, the results continue to hold provided that $1-p-q$ is sufficiently small. That is, there exists a threshold $\eta > 0$, such that the result holds provided the sufficient condition $1-p-q < \eta$.

Step 2: We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed investors, $\frac{d\Theta_t^*(m)}{dn_t}$:

$$\frac{d\Theta_t^*(n,-s\Delta)}{dn_t} = \frac{\begin{vmatrix} -\frac{\partial K_{1,2}}{\partial n_t} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,0)} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,\Delta)} \\ -\frac{\partial K_3}{\partial n_t} & \frac{\partial K_3}{\partial \Theta_t(n_t,0)} & \frac{\partial K_3}{\partial \Theta_t(n_t,\Delta)} \\ -\frac{\partial L}{\partial n_t} & \frac{\partial L}{\partial \Theta_t(n_t,0)} & \frac{\partial L}{\partial \Theta_t(n_t,\Delta)} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,-s\Delta)} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,0)} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,\Delta)} \\ \frac{\partial K_3}{\partial \Theta_t(n_t,-s\Delta)} & \frac{\partial K_3}{\partial \Theta_t(n_t,0)} & \frac{\partial K_3}{\partial \Theta_t(n_t,\Delta)} \\ \frac{\partial L}{\partial \Theta_t(n_t,-s\Delta)} & \frac{\partial L}{\partial \Theta_t(n_t,0)} & \frac{\partial L}{\partial \Theta_t(n_t,\Delta)} \end{vmatrix}} \equiv \frac{|M_1|}{|M|} \tag{34}$$

where $|M| \equiv \det(M)$. Similarly we can derive $\frac{d\Theta_t^*(n,0)}{dn_t} = \frac{|M_2|}{|M|}$ and $\frac{d\Theta_t^*(n,\Delta)}{dn_t} = \frac{|M_3|}{|M|}$.

To find $|M|$, recall from the proof of Proposition 4 that $\frac{\partial K_1}{\partial \Theta_t(0)} > 0$, $\frac{\partial K_1}{\partial \Theta_t(-s\Delta)} < 0$ and $\frac{\partial K_1}{\partial \Theta_t(\Delta)} = 0$, while $\frac{\partial K_2}{\partial \Theta_t(0)} > 0$, $\frac{\partial K_2}{\partial \Theta_t(-s\Delta)} = 0$ and $\frac{\partial K_2}{\partial \Theta_t(\Delta)} < 0$. Furthermore, $\frac{\partial K_3}{\partial \Theta_t(0)} = 0$, $\frac{\partial K_3}{\partial \Theta_t(\Delta)} < 0$ and $\frac{\partial K_3}{\partial \Theta_t(-s\Delta)} > 0$. Finally, $\frac{\partial L}{\partial \Theta_t(m)} < 0\ \forall m$ for a sufficiently high but finite value of β. As a result, $|M| > 0$ for a sufficiently high but finite value of β, irrespective of which of the two systems is used. That is, there exists a threshold $\beta > 0$, such that the result holds provided the sufficient condition $\beta > \underline{\beta}$.

The proof proceeds by analyzing $|M_1|$, $|M_2|$, and $|M_3|$. To do this, we first examine the derivatives $\frac{\partial K_1}{\partial n_t}$, $\frac{\partial K_3}{\partial n_t}$ and $\frac{\partial L}{\partial n_t}$. Thereafter, we combine the results to obtain the signs of the determinants $\frac{\partial K_1}{\partial n_t} = \frac{\partial x_{tU}(0)}{\partial n_t} - \frac{\partial x_{tU}(-s\Delta)}{\partial n_t}$, $\frac{\partial K_2}{\partial n_t} = \frac{\partial x_{tU}(0)}{\partial n_t} - \frac{\partial x_{tU}(\Delta)}{\partial n_t}$ and $\frac{\partial K_3}{\partial n_t} = \frac{\partial x_{tU}(-s\Delta)}{\partial n_t} - \frac{\partial x_{tU}(\Delta)}{\partial n_t}$, where:

$$\frac{\partial x_{tU}(m)}{\partial n_t} \equiv \frac{\Theta_t(m) - \Phi\left(\frac{\alpha(\Theta_t(m)-(\mu+m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right)}{(1-n_t)^2\phi(\Phi^{-1}(\cdot))}. \tag{35}$$

To evaluate this partial derivatives, we can use the optimality condition in the case of symmetrically informed investors, $n_t = 1$. That is, $\Theta_t^*(1,m)$ is defined as the solution to $F_t(\Theta_t^*(1,m),m) = 0$, where uniqueness requires that F_t is strictly decreasing in the first argument. This implies:

$$\Theta_t(m) - \Phi\left(\frac{\alpha(\Theta_t(m)-(\mu+m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right) \lesseqqgtr 0\ if\ \Theta_t(m) \lesseqqgtr \Theta_t(1,m).$$

There are four cases. Case 1: $\Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,-s\Delta) \leq \Theta_t^*(1,-s\Delta))$. Case 2: $\Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(1,0) \leq$

$\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,-s\Delta))$. Case 3: $\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta)$. Case 4: $\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,0) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta)$.

Case 1: Using K_1 and K_3 we obtain $\frac{\partial K_1}{\partial n} > 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} < 0 \,\forall\, n_t \in [0,1)$.

Case 2: Using K_2 and K_3 we obtain $\frac{\partial K_2}{\partial n} < 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} < 0 \,\forall\, n_t \in [0,1)$.

Case 3: Using K_1 and K_3 we obtain $\frac{\partial K_1}{\partial n} > 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} > 0 \,\forall\, n_t \in [0,1)$.

Case 4: Using K_2 and K_3 we obtain $\frac{\partial K_2}{\partial n} < 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} > 0 \,\forall\, n_t \in [0,1)$.

After having found the partial derivative for first two equilibrium conditions ($K_{1,2}$), we turn to the other equilibrium condition (L). Here, we can invoke the envelope theorem in order to obtain $\frac{\partial L}{\partial n} = 0$. The idea is the following. Since L represents the indifference condition of an uninformed investor, the proportion of informed investors enters only indirectly via x_{tU}^* and we can write:

$$\frac{\partial L}{\partial n} = \frac{\partial J}{\partial x_{tU}^*}\frac{\partial x_{tU}^*}{\partial n} + \overbrace{\frac{\partial J}{\partial n}}^{=0}. \tag{36}$$

Since x_{tU}^* is the optimal signal threshold of an uninformed investor, it satisfies $J(\cdot, x_{tU}^*) = \gamma_t$. Thus, we must have $\frac{\partial J}{\partial x_{tU}^*} = 0$, which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

To conclude, we have for all cases that $|M| > 0$ provided that $\beta > \underline{\beta}$. It shows that $|M_1| > 0$ for case 1 and $|M_3| < 0$ for case 2, while $|M_1| < 0$ for case 1 and $|M_3| > 0$ for case 2. Furthermore, for the probability of $m = 0$, i.e. $1 - p - q$, sufficiently small we have that $|M_1| > 0$ also for case 2 and $|M_3| < 0$ also for case 1, while $|M_1| < 0$ also for case 2 and $|M_3| > 0$ also for case 1. Hence, provided that $1 - p - q < \eta$ and $\beta > \underline{\beta}$, we find $\forall\, n_t \in [0,1)$:

$$\frac{d\Theta_t^*(n_t,-s\Delta)}{dn_t} = \begin{cases} > 0 & if\ \Theta_t^*(n_t,-s\Delta) < \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) > \Theta_t^*(1,\Delta) \\ < 0 & if\ \Theta_t^*(n_t,-s\Delta) > \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) < \Theta_t^*(1,\Delta) \\ = 0 & if\ \Theta_t^*(n_t,-s\Delta) = \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) = \Theta_t^*(1,\Delta) \end{cases}$$

and $\forall n_t \in [0,1)$:

$$\frac{d\Theta_t^*(n_t,\Delta)}{dn_t} = \begin{cases} < 0 \quad if\ \Theta_t^*(n_t,-s\Delta) < \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) > \Theta_t^*(1,\Delta) \\ > 0 \quad if\ \Theta_t^*(n_t,-s\Delta) > \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) < \Theta_t^*(1,\Delta) \\ = 0 \quad if\ \Theta_t^*(n_t,-s\Delta) = \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) = \Theta_t^*(1,\Delta) \end{cases}$$

Step 3: In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold w.r.t. the proportion of informed investors is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed investors are always bounded, which proves Result (A). Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed, yielding Result (B). That is, given the result from *step 1*, the second line of each derivative drops and equations (30) and (31) follow.

We prove that $\Theta_t^*(1,\Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1,-s\Delta)$ for all n_t if α sufficiently small. First, $\Theta_t^*(1,\Delta) < \Theta_t^*(\Delta) = \Theta_t^*(0) = \Theta_t^*(-s\Delta) < \Theta_t^*(1,-s\Delta)$ if $n_t = 0$, while $\Theta_t^*(1,\Delta) = \Theta_t^*(\Delta)$ and $\Theta_t^*(1,-s\Delta) = \Theta_t^*(-s\Delta)$ if $n_t = 1$. Second, $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=0} < 0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=0} > 0$ and $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=1} = \frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=1} = 0$. Third, by continuity $\Theta_t^*(1,\Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1,-s\Delta)$ and $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=0} < 0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=0} > 0$ for small values of n_t. Fourth, if for any $\hat{n}_t \in (0,1]$ $\Theta_t^*(-s\Delta) \nearrow \Theta_t^*(1,-s\Delta)$ when $n_t \to \hat{n}_t$, then – for sufficiently small but positive values of α – it has to be true that $\Theta_t^*(\Delta) \searrow \Theta_t^*(1,\Delta)$ when $n_t \to \hat{n}_t$. This is because of the result in *step 1*. Fifth, given that the derivatives of the fundamental thresholds flip when both are outside of the bounds we have $\Theta_t^*(1,\Delta) = \Theta_t^*(\Delta)$ and $\Theta_t^*(1,-s\Delta) = \Theta_t^*(-s\Delta)$ for all $n_t \geq \hat{n}_t$. In conclusion, $\Theta_t^*(1,\Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1,-s\Delta)$ for all $n_t \in [0,1]$ if α sufficiently small.

Result (C). From the indifference conditions for informed investors:

$$\frac{dx_{tI}^*(m)}{dn_t} = \frac{d\Theta_t^*(m)}{dn_t}\left(\frac{\beta}{\alpha+\beta}\right)^{-1}. \tag{37}$$

Therefore, by continuity, there exists a sufficiently small but positive value of α, say $\overline{\alpha}$, that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. The distance between the fundamental thresholds is monotone for any $n_t > 0$, which implies

$\frac{d(x^*_{tI}(n_t,-s\Delta)-x^*_{tI}(n_t,\Delta))}{dn_t} \geq 0 \;\; \forall\, n_t \in [0,1)$. Furthermore, $x^*_{tI}(n_t,-s\Delta) - x^*_{tI}(n_t,\Delta)) > 0 \;\; \forall\, n_t \in [0,1]$.

Result (D). We proceed in two steps.

Step 1: We solve the equilibrium condition in equation (26) for $n_2 = 0$ to show that $[\Theta^*(n_2,m)|f = 1] > [\Theta^*(n_2,m)|f = 0]$, $\forall m \in \{\Delta, -s\Delta, 0\}$, building on the results of Lemmas 1 and 2.

Step 2: We next inspect the equilibrium adjustment of the signal thresholds. First, recall from Lemma 2 that, for sufficiently high values of β, a higher $\Theta^*_I(0,m)$ implies a higher $x^*_U(0,m)$. Second, we consider the marginal investor who becomes informed. From equation (22):

$$\Phi\big([\Theta^*_I(0,m)|f]\sqrt{\alpha+\beta} - \frac{\alpha(\mu+m)+\beta[x^*_I(n_2,m)|f]}{\sqrt{\alpha+\beta}}\big) = \gamma_2. \tag{38}$$

The result in *Step 1* implies that inequality (33) follows. This completes the proof.

A.2.3 Information stage in region 2: strategic complementarity in information choices

We next study the value of information about the macro shock. The value of information to an individual investor is defined as the difference in the expected utility between an informed and an uninformed investor before costs. These expected utilities are denoted by EU_I and EU_U, respectively. The expected utility of an informed investor writes:

$$\begin{aligned}
&\mathbb{E}[u(d_i = I, n_2)] \equiv EU_I - c \\
= \quad & -c + p' \begin{pmatrix} \int_{-\infty}^{\Theta^*_2(n_2,\Delta)} b_t \int_{x_{i2}\leq x^*_I(n_2,\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta^*_2(n_2,\Delta)}^{+\infty} \ell_2 \int_{x_{i2}\leq x^*_I(n_2,\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix} + \\
& q' \begin{pmatrix} \int_{-\infty}^{\Theta^*_2(n_2,-s\Delta)} b_2 \int_{x_{i2}\leq x^*_I(n_2,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|-s\Delta)d\Theta_2 \\ -\int_{\Theta^*_2(n_2,-s\Delta)}^{+\infty} \ell_2 \int_{x_{i2}\leq x^*_I(n_2,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|-s\Delta)d\Theta_2 \end{pmatrix} + \\
& (1-p'-q') \begin{pmatrix} \int_{-\infty}^{\Theta^*_2(n_2,0)} b_2 \int_{x_{i2}\leq x^*_I(n_2,0)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|0)d\Theta_2 \\ -\int_{\Theta^*_2(n_2,0)}^{+\infty} \ell_2 \int_{x_{i2}\leq x^*_I(n_2,0)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|0)d\Theta_2 \end{pmatrix},
\end{aligned} \tag{39}$$

By contrast, the expected utility of an uninformed investor, $E[u(d_i = U, n_2)] \equiv EU_U$, is constructed in the same way as EU_I with the difference that all signal thresholds have to be replaced by $x^*_U(n_2)$.

Let $v \equiv EU_I - EU_U$ be the value of information conditional on the proportion of informed investors and the information set in region 2:

$$v(n_2) = \quad p'\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,\Delta)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,\Delta)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \end{array}\right)$$
$$+q'\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \end{array}\right)$$
$$+(1-p'-q')\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,0)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,0)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|0)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,0)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,0)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|0)d\theta_2 \end{array}\right). \tag{40}$$

The distribution of the fundamental conditional on the realized macro shock, $f(\Theta_2|m)$, is normal with mean $\mu+m$ and precision α. The distribution of the private signal conditional on the fundamental, $g(x|\Theta_2)$, is normal with mean Θ_2 and precision β.

To build intuition, suppose that $1-p-q\to 0$. Given $\Theta_2^*(1,-s\Delta) > \Theta_2^*(1,\Delta)$ we have that $x_I^*(n_2,-s\Delta) > x_U^*(n_2) > x_I^*(n_2,\Delta)$ and marginal benefit of increasing $x_I^*(n_2,-s\Delta)$ above $x_U^*(n_2)$ is:

$$p'\left(\begin{array}{c} b_2\int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} g(x_U^*|\Theta_2)f(\Theta_2)d\Theta_2 \\ -\ell_2\int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} g(x_U^*|\Theta_2)f(\Theta_2)d\Theta_2 \end{array}\right) > 0, \tag{41}$$

while the marginal benefit of increasing $x_I^*(n_2,\Delta)$ above $x_U^*(n_2)$ is:

$$q'\left(\begin{array}{c} b_2\int_{-\infty}^{\Theta_2^*(n_2,\Delta)} g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \\ -\ell_2\int_{\Theta_2^*(n_2,\Delta)}^{+\infty} g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \end{array}\right) < 0. \tag{42}$$

These expressions are best understood in terms of type-I and type-II errors. Each of the expressions in equations (41) and (42) have two components. The first component in each equation represents the marginal benefit of attacking when a crisis occurs. Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error). The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 3 together with Corollary 4 imply the following. The marginal benefit of increasing $x_I^*(n_2,-s\Delta)$ above $x_U^*(n_2)$ is positive because the type-I error is relatively more costly than the type-II error. By contrast, the marginal benefit of decreasing $x_I^*(n_2,\Delta)$ below $x_U^*(n_2)$ is positive because the type-II error is more costly. In sum, informed investors attack more aggressively upon learning that $m = -s\Delta$ and less aggressively upon learning $m = \Delta$. The value of information is governed by the relationship between the type-I and type-II errors. When the signal thresholds of informed and uninformed investors differ, the value of information is positive because the difference in thresholds increases in the proportion of informed investors. The result in Lemma 4 follows.

Lemma 4 ***Strategic complementarity in information choices.*** *If Assumption 1 holds, the value of information increases in the proportion of informed investors:*

$$\frac{dv(n_2, f)}{dn_2} \geq 0, \tag{43}$$

with strict inequality for small values of n_2.

Proof Under the sufficient conditions of Assumption 1 we have that $\Theta_2^*(n_2, -s\Delta) > \Theta_2^*(n_2, \Delta)$ and $x_I^*(n_2, -s\Delta) > x_U^*(n_2, 0) > x_I^*(n_2, \Delta)$. We will prove that $\frac{dv(n_2, f)}{dn_2} \geq 0$ and $v(n_2, f) > 0 \ \forall\, n_2 \in (0,1] \wedge f \in \{0,1\}$. Suppose that $1 - q - p \to 0$, then the last term of $\mathbb{E}[u(d_i = I, n_2)]$ and $\mathbb{E}[u(d_i = U, n_2)]$ vanishes. Given that $\Theta_2^*(n_2, -s\Delta) > \Theta_2^*(n_2, \Delta)$, the first two summands of equation (40) are strictly positive and, hence, $v(n_2) > 0 \ \ \forall\, n_2 \in (0,1]$. Furthermore, given Lemma 3, an increase in the proportion of informed investors is associated with a (weak) increase in both $\Theta_2^*(n_2, -s\Delta)$ and $x_I^*(n_2, -s\Delta)$ as well as a (weak) decrease in both $\Theta_2^*(n_2, \Delta)$ and $x_I^*(n_2, \Delta)$. For a given x_U^*, an increase in n_2 leads to a relative increase of the (positive) loss component in the first summand of equation (40) and a relative increase of the benefit component in the second summand. By continuity and monotonicity, any general equilibrium adjustment of $x_U^*(n_2)$ with n_2 cannot fully off-set the previous effects. For this reason, the left-hand side of equation (40) increases in n_2. Thus, $\frac{dv(n_2, f)}{dn_2} \geq 0$. By continuity, the results continue to hold if $1 - p - q$ is sufficiently small, that is if $1 - p - q < \eta$. This concludes the proof.

A.2.4 Proof of Proposition 2

We prove the results of the inequalities in (7). Given Assumption 1, the results of Lemma 4 apply and the first and third inequality follow. The proof of the second inequality consists of four steps.

Step 1: Suppose that $1 - p - q \to 0$ and evaluate equation (40) at $n_2 = 1$. First, observe that the first term in brackets is only affected by s through $x_U^*(1)$. Second, observe that the second term in brackets is growing strictly larger in s for a given $x_U^*(1)$, as $x_I^*(1, -s\Delta)$ grows in s because of the indifference condition of informed investors. Third, if $f=0$ observe that $x^*_U(1)\to x^*_I(1,\Delta)$ as $s\to\infty$. Given that the term in in the second bracket is finite and multiplied by $q=\frac{p}{s}$, we have that $v(1,f=0)>v(0,f=0)\to 0$ for $s\to\infty$, where the inequality is due to the result in Lemma 4.

Step 2: Now, suppose that $f=1$ and note that:

$$\lim_{\mu\to\infty}[q'|f=1]\big|_{s=\frac{\mu}{\Delta}+\iota}=\lim_{\mu\to\infty}\frac{\frac{p}{s}\Phi\big(\sqrt{\alpha_1}(\Theta^*_1(\mu)-\mu+s\Delta)\big)}{\begin{pmatrix}p\Phi\big(\sqrt{\alpha_1}(\Theta^*_1(\mu)-\mu-\Delta)\big)\\+\frac{p}{s}\Phi\big(\sqrt{\alpha_1}(\Theta^*_1(\mu)-\mu+s\Delta)\big)\\+(1-p-\frac{p}{s})\Phi\big(\sqrt{\alpha_1}(\Theta^*_1(\mu)-\mu)\big)\end{pmatrix}}\Big|_{s=\frac{\mu}{\Delta}+\iota}=1,$$

where $s=\frac{\mu}{\Delta}+\iota$ with $\iota>0$ is necessary to maintain the assumption that the prior is weak after observing a negative macro shock. Conversely, for $f=0$ we have $\lim_{\mu\to\infty}[q'|f=0]\big|_{s=\frac{\mu}{\Delta}+\iota}=0$.

Step 3: Next, notice that for a given μ and $s>1$, the event of a negative macro shock is never considered to be the most probable state of the world provided that s is sufficiently high. This is because $q'<p'$ holds for finite μ if s is sufficiently high: $s\geq \Pr\{f|m=-s\Delta\}(\Pr\{f|m=\Delta\})^{-1},\forall f\in\{0,1\}$. Moreover, given *step 2* we have that $[q'|f=1]>>0$ for μ sufficiently high such that $[p'|f=1]>[q'|f=1]>0$, provided s is sufficiently high as well; and in the limit approaching ∞ with a higher speed of convergence. Instead, $[q'|f=0]$ is arbitrarily small.

Step 4: Given the comparative statics in *step 3*, we have for sufficiently high values of s and μ that the there is a strictly positive probability weight on the first and second bracket of $v(1,1)$, while all the probability weight is concentrated on the first bracket of $v(1,0)$. In addition, the expression in the first bracket of $v(1,1)$ is strictly larger than the expression in the first bracket of $v(1,0)$ since $x^*_U\nrightarrow x^*_I(1,\Delta)$ and $x^*_U\nrightarrow x^*_I(1,-s\Delta)$ in the former case, while $x^*_U\to x^*_I(1,\Delta)$ in the latter case. In fact, both expressions approach zero for $\mu\to\infty$, but the expression in the first bracket of $v(1,0)$ approaches zero with a higher speed of convergence. Conversely, the expression in the second bracket of $v(1,1)$ is potentially smaller than the expression in the second bracket of $v(1,0)$. Both terms approach zero with the same speed of convergence for $\mu\to\infty$. The pre-multiplied conditional probabilities $[q'|f=1]$ and $[q'|f=0]$ make the difference, where we have:

$$\lim_{\mu\to\infty}[q'|f=0]\big|_{s=\frac{\mu}{\Delta}+\iota} = \lim_{\mu\to\infty} \frac{\frac{p}{s}\left(1-\Phi\left(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu+s\Delta)\right)\right)}{\begin{pmatrix} 1-p\Phi\left(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu-\Delta)\right) \\ -\frac{p}{s}\Phi\left(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu+s\Delta)\right) \\ -(1-p-\frac{p}{s})\Phi\left(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu)\right) \end{pmatrix}}\Big|_{s=\frac{\mu}{\Delta}+\iota} = 0.$$

In the limit $[q'|f=1]/[q'|f=0]\to\infty$ for $\mu\to\infty$ and s sufficiently high such that $0<[q'|f=1]<1$. Taken together, $v(1,1)-v(1,0)>0$ in the limit since the other terms in brackets approach zero with the same speed of convergence. By continuity, the result also holds for large, but finite, values of s and μ, as well as for sufficiently small $1-p-q$. Hence, $v(n_2=0,f=1)>v(n_2=1,f=0)$ and inequality (7) follows provided that Assumption 1 holds and s and μ are sufficiently high.

A.2.5 Proof of Proposition 1

The proof builds on the analysis of the coordination and information stages in region 2. Corollary 4 establishes the existence of unique attack rules in region 2. Proposition 2 establishes the existence of a nonempty intermediate range of information costs $c\in(\underline{c},\overline{c})$ with $\underline{c}\equiv v(1,0)$ and $\overline{c}\equiv v(0,1)$, such that all investors choose to acquire information if and only if a crisis occurs in region 1. The result in Proposition 1 follows.

A.3 Proof of Proposition 3

The proof consists of four steps. First, suppose that $s\to\infty$. Not observing a crisis in region 1 implies that $\frac{q'}{q}\to 0$ as q' goes to zero faster than q. To see this, observe that $\Pr\{f=0|m=-s\Delta\}\to 0$ if $s\to\infty$, since a Θ_2 drawn from a distribution with a highly negative mean, $\mu-s\Delta$, is increasingly unlikely to have a sufficiently high realization such that $f=0$ occurs. At the same time, $\frac{p'}{p}\nrightarrow 0$ and $\frac{1-p'-q'}{1-p-q}\nrightarrow 0$ if $s\to\infty$ and $f=0$.

Second, the right-hand side of inequality (8) has a fundamental threshold that is lower than the fundamental threshold on the left-hand side. To see this, we again use the comparative static result underlying Lemma 2. Observing $f=0$ implies that the second summand of J in equation (21) goes to zero if $s\to\infty$. Hence, $[\Theta_2^*(n_2=0,m)|f=0]<[\Theta_2^*(n_2=1,m=0)|f=1]$.

Third, given $s \to \infty$, the Θ's on the right-hand side of inequality (8) are drawn from equally favorable or, with a positive probability ($\frac{p'}{p} \nrightarrow 0$) that is away from zero, from a more favorable distribution if $f = 0$. Thus, the likelihood of a crisis in region 2 is lower if $f = 0$ and $s \to \infty$.

Fourth, by continuity, the result can be generalized to hold for a sufficiently high, but finite, value of s, say $s > \underline{s}$. This concludes the proof.

A.4 Extension of the contagion result

The purpose of this extension is to establish that contagion arises from an ex-ante perspective. The analysis is conducted for a special case, but a numerical analysis suggests that the result in Proposition 6 holds for a large range of parameters, including for those used in Figures 1 and 2.

To simplify, we start from the case $\mu = \gamma = \frac{1}{2}$ and $s = 1$, where the equilibrium thresholds are symmetric (because of the properties of the Gaussian distribution). This approach allows us to analytically examine for $s \searrow 1$ how the ex-ante probability of a crisis in region 2 absent the learning about region 1 (defined below) changes in s.

Proposition 6 ***Ex-ante benchmark.*** *Let $\mu = \gamma = \frac{1}{2}$ and $s \searrow 1$. A financial crisis in region 2 is more likely after a crisis in region 1 when all investors learn that the macro shock is zero, than the ex-ante probability of a crisis in region 2 absent the learning about region 1:*

$$\begin{aligned}
&\Pr\{\Theta_2 < \Theta_I^*(m)|m=0\} > \qquad (44)\\
&P_0 \equiv p\Pr\{\Theta_2 < \Theta_2^*|m=\Delta\} + q\Pr\{\Theta_2 < \Theta_2^*|m=-s\Delta\} + (1-p-q)\Pr\{\Theta_2 < \Theta_2^*|m=0\},
\end{aligned}$$

where $\Theta_1^* = \Theta_2^*$ solves equation (14).

Proof Suppose that $\mu = \gamma = \frac{1}{2}$ and $s = 1$. Then $\Pr\{\Theta_2 < \Theta_I^*(m)|m=0\} = \frac{1}{2} = P_0 = \frac{1}{2}$. Moreover, $d\Pr\{\Theta_2 < \Theta_I^*(m)|m=0\}/ds = 0$ for $s \searrow 1$. What remains is to analyse dP_0/ds which, if negative, confirms inequality (44).

$$\begin{aligned}
\frac{dP_o}{ds}|_{\mu=\gamma=\frac{1}{2},s=1} \;=\;& \left(p\phi\left(\sqrt{\alpha}(-\Delta)\right) + q\phi\left(\sqrt{\alpha}\Delta\right) + (1-p-q)\phi\left(0\right)\right)\sqrt{\alpha}\frac{d\Theta_2^*}{ds}|_{\mu=\gamma=\frac{1}{2},s=1}\\
&+p\left(\Phi(0) - \Phi(\sqrt{\alpha_2}\Delta)\right).
\end{aligned}$$

The second summand is strictly negative. For the first summand we need to inspect the derivative of the fundamental threshold $d\Theta_2^*/ds = -(dJ(\Theta_2^*,s)/ds)/(dJ(\Theta_2^*,s)/d\Theta_2^*)$, where $J(\Theta_2^*,s)$ is the solution to equation (14) after plugging in for the signal threshold. We first analyze $dJ(\Theta_2^*,s)/d\Theta_2^*$.

$$\begin{aligned}\frac{dJ(\Theta_2^*,s)}{d\Theta_2^*}|_{\mu=\gamma=\frac{1}{2},s=1} &= [\Phi\left(\frac{-\alpha\Delta}{\sqrt{\alpha+\beta}}\right)-\Phi(0)]\frac{d\hat{p}^*}{d\Theta_2^*}|_{\mu=\gamma=\frac{1}{2},s=1}+[\Phi\left(\frac{\alpha\Delta}{\sqrt{\alpha+\beta}}\right)-\Phi(0)]\frac{d\hat{q}^*}{d\Theta_2^*}|_{\mu=\gamma=\frac{1}{2},s=1}\\ &+\frac{\alpha-\sqrt{\beta}/\phi(0)}{\sqrt{\alpha+\beta}}\left(\begin{array}{c}\phi(-\sqrt{\alpha}\Delta)\hat{p}^*|_{\mu=\gamma=\frac{1}{2},s=1}+\phi(\sqrt{\alpha}\Delta)\hat{q}^*|_{\mu=\gamma=\frac{1}{2},s=1}\\ +\phi(0)(1-\hat{p}^*-\hat{q}^*)|_{\mu=\gamma=\frac{1}{2},s=1}\end{array}\right)<0,\end{aligned}$$

where the first two summands are exactly off-setting each other for $\mu=\gamma=\frac{1}{2}$ and $s=1$, where $\frac{dx_2^*}{d\Theta_2^*}=1$. Moreover, the third summand is finite and strictly negative provided β is sufficiently high to ensure the existence of a unique threshold as guaranteed by Assumption 1. We next examine:

$$\begin{aligned}\frac{dJ(\Theta_2^*,s)}{ds}|_{\mu=\gamma=\frac{1}{2},s=1} &= [\Phi\left(\frac{-\alpha\Delta}{\sqrt{\alpha+\beta}}\right)-\Phi(0)]\frac{d\hat{p}^*}{ds}|_{\mu=\gamma=\frac{1}{2},s=1}+[\Phi\left(\frac{\alpha\Delta}{\sqrt{\alpha+\beta}}\right)-\Phi(0)]\frac{d\hat{q}^*}{ds}|_{\mu=\gamma=\frac{1}{2},s=1}\\ &+\frac{\alpha\Delta}{\sqrt{\alpha+\beta}}\phi\left(\frac{\alpha\Delta}{\sqrt{\alpha+\beta}}\right)\hat{q}^*|_{\mu=\gamma=\frac{1}{2},s=1},\end{aligned}$$

and find that $dJ(\Theta_2^*,s)/ds$ is arbitrarily close to zero for large values of β. As a result, $dP_0/ds<0$ for $\mu=\gamma=\frac{1}{2}$, $s\searrow 1$ and sufficiently high β as in Assumption 1. This concludes the proof.

A.5 Proof of Corollary 1

This proof consists of five steps. We first demonstrate in *Step 1* that $\Pr\{\Theta_2<\Theta_I^*|m=0,f=1\}>\Pr\{\Theta_2<\Theta_U^*|f=0\}$, $\forall p,\Delta$. Thereafter, we show that $\frac{d\Pr\{\Theta_2<\Theta_U^*|f=0\}}{dp}<0$. Given that $\Pr\{\Theta_2<\Theta_I^*|m=0,f=1\}$ is invariant in p, we can establish equation (9). To do so, we consider the special case where $\mu=\gamma=\frac{1}{2}$ as in Figure 3. Since $s>1$ is only key for the differential information choice, but not for the Bayesian updating channel, we further simplify by considering the case where $s=1$.

Step 1: With $\mu=\gamma=\frac{1}{2}$ and $s\to 1$ we have due to symmetry that $\Theta_U^*=x_U^*=\frac{1}{2}$ and $p=q$. Form Lemma 2 we have that $p'>q'$ if a crisis in region 1 is not observed, $f=0$. As a result, ceteris paribus, $\hat{p}'>\hat{q}'$. From the equilibrium condition in equation (21) we can prove by contradiction that $\Theta_U^*<\frac{1}{2}$ if $p'>q'$. Hence, $\Pr\{\Theta_2<\Theta_I^*|m=0,f=1\}=\frac{1}{2}>\Pr\{\Theta_2<\Theta_U^*|f=0\}$, $\forall p,\Delta$.

Step 2: An increase in p has the following implications:

$$\begin{aligned}
\frac{dPr\{\Theta_2 < \Theta_U^*|f=0\}}{dp} &= \frac{dp'}{dp}Pr\{\Theta_2 < \Theta_U^*|m=\Delta\} + p'\frac{dPr\{\Theta_2 < \Theta_U^*|m=\Delta\}}{dp} \\
&+ \frac{dq'}{dp}Pr\{\Theta_2 < \Theta_U^*|m=-s\Delta\} + q'\frac{dPr\{\Theta_2 < \Theta_U^*|m=-s\Delta\}}{dp} \\
&- \left(\frac{dp'}{dp}+\frac{dq'}{dp}\right)Pr\{\Theta_2 < \Theta_U^*|m=0\} + (1-p'-q')\frac{dPr\{\Theta_2 < \Theta_U^*|m=0\}}{dp}.
\end{aligned} \tag{45}$$

Step 3: We first inspect $\frac{dp'}{dp}$, noting that the symmetry property prevails when changing p in equation (14) so that Θ_1^* is unaltered and $\frac{d\Pr\{f=1|m\}|_{s=1}}{dp} < 0$ and $\frac{d\Pr\{f=0|m\}|_{s=1}}{dp} > 0, \forall m \in \{\Delta, -s\Delta, 0\}$:

$$\begin{aligned}
\left.\frac{dp'}{dp}\right|_{s=1} &= \left.\frac{\left(\Pr\{f=0|m=\Delta\} + p\frac{d\Pr\{f=0|m=\Delta\}}{dp}\right)\Gamma_2(f=0) - p\Pr\{f=0|m=\Delta\}\frac{d\Gamma_2(f=0)}{dp}}{\Gamma_2^2(f=0)}\right|_{s=1} \\
\left.\frac{d\Gamma_2(f=0)}{dp}\right|_{s=1} &= \left.\begin{pmatrix} \Pr\{f=0|m=\Delta\} + p\frac{d\Pr\{f=0|m=\Delta\}}{dp} \\ +\frac{1}{s}\Pr\{f=0|m=-s\Delta\} + q\frac{d\Pr\{f=0|m=-s\Delta\}}{dp} \\ -\left(1+\frac{1}{s}\right)\Pr\{f=0|m=0\} + (1-p-q)\frac{d\Pr\{f=0|m=0\}}{dp} \end{pmatrix}\right|_{s=1} > 0.
\end{aligned}$$

Observe that $\frac{d\Pr\{f=0|m\}}{dp} = 0$ if $\alpha_1 \to 0$. Doing the same for $\frac{dq'}{dp}$ we can show that $\frac{dp'|_{s=1}}{dp} > \frac{dq'|_{s=1}}{dp} > 0$ if α_1 is sufficiently small and $\lim_{\alpha_1 \to 0}\left(\frac{dp'}{dp}\big|_{s=1}\right) = \lim_{\alpha_1 \to 0}\left(\frac{dq'}{dp}\big|_{s=1}\right) > 0$.

Step 4: Next, we inspect $sign\left(\frac{dPr\{\Theta_2 < \Theta_U^*|m\}}{dp}\right) = sign\left(\frac{d\Theta_U^*}{dp}\right)$ by analyzing the equilibrium condition in equation (20), which leads to $\frac{d\Theta_U^*}{dp} < 0$ provided p is sufficiently small. In fact, $\lim_{p\to 0}\Theta_U^* = x_U^* = \frac{1}{2}$ and $\lim_{p \to 0}\frac{d\Theta_U^*}{dp} < 0$. To see this, we apply the implicit function theorem to equation (21):

$$\frac{d\Theta_U^*}{dp} = -\frac{dJ(0,\Theta_2^*(-s\Delta),\Theta_2^*(\Delta),\Theta_2^*(0))/dp}{dJ(0,\Theta_2^*(-s\Delta),\Theta_2^*(\Delta),\Theta_2^*(0))/d\Theta_U^*}.$$

From Corollary 4 we know that $\frac{dL(\cdot)}{d\Theta_U^*} < 0$. Moreover:

$$\begin{aligned}
\left.\frac{dJ(\cdot)}{dp}\right|_{s=1} &= \frac{dp'}{dp}\frac{\Pr\{x_U^*|m=\Delta\}}{\Gamma_1}\Psi(\Theta_U^*, x_U^*, \Delta)\big|_{s=1} + \frac{dq'}{dp}\frac{\Pr\{x_U^*|m=-s\Delta\}}{\Gamma_1}\big|_{s=1}\Psi(\Theta_U^*, x_U^*, -s\Delta) \\
&- \left(\frac{dp'}{dp}+\frac{dq'}{dp}\right)\frac{\Pr\{x_U^*|m=0\}}{\Gamma_1}\Psi(\Theta_U^*, x_U^*, 0)\big|_{s=1}
\end{aligned}$$

is strictly negative and away from zero for sufficiently small values of α_1.

Step 5: Inspecting equation (45), there exists, by continuity, a sufficiently small positive value of p and α_1 such that $\frac{dPr\{\Theta_2<\Theta_U^*|f=0\}}{dp}<0$ and $\frac{d\Pr\{\Theta_2<\Theta_I^*(m)|m=0,f=1\}-\Pr\{\Theta_2<\Theta_U^*|f=0\}}{d(1-p-q)}<0$. This concludes the proof.

A.6 Proof of Corollary 2

We wish to show that $\frac{dv(0,f)}{d(1-p-q)}<0$. Note that $sign\left(\frac{dv(0,f)}{d(1-p-q)}\right)=-sign\left(\frac{dv(0,f)}{dp}\right)$ and $sign\left(\frac{dp'}{d(1-p-q)}\right)=-sign\left(\frac{dp'}{dp}\right)$ since $q=\frac{p}{s}$. The derivative of the value of information can be derived as:

$$\begin{aligned}
\frac{dv(0,1)}{dp} \;=\; & \frac{dp'}{dp}\begin{pmatrix} \int_{\Theta_2^*(0,m)}^{+\infty}\ell_2\int_{x_I^*(0,\Delta)}^{x_U^*(0)}g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{-\infty}^{\Theta_2^*(0,m)}b_2\int_{x_I^*(0,\Delta)}^{x_U^*(0)}g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix} \\
& +\frac{dq'}{dp}\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(0,m)}b_2\int_{x_U^*(0)}^{x_I^*(0,-s\Delta)}g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(0,m)}^{+\infty}\ell_2\int_{x_U^*(0)}^{x_I^*(0,-s\Delta)}g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \end{pmatrix} \\
& -\left(\frac{dp'}{dp}+\frac{dq'}{dp}\right)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(0,m)}b_2\int_{x_U^*(0)}^{x_I^*(0,0)}g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|0)d\Theta_2 \\ +\int_{\Theta_2^*(0,m)}^{+\infty}\ell_2\int_{x_U^*(0)}^{x_I^*(0,0)}g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|0)d\Theta_2 \end{pmatrix} \\
& +\frac{d\Theta_2^*(0,m)}{dp}(b_2+\ell_2)\begin{pmatrix} -p'\int_{x_I^*(0,\Delta)}^{x_U^*(0)}g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|\Delta) \\ +q'\int_{x_U^*(0)}^{x_I^*(0,-s\Delta)}g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|\Delta) \\ +(1-p'-q')\int_{x_I^*(0,0)}^{x_U^*(0)}g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|\Delta) \end{pmatrix} \\
& +\frac{dx_U^*(0)}{dp}\begin{pmatrix} p'\left(\int_{\Theta_2^*(0,m)}^{+\infty}\ell_2 g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2-\int_{-\infty}^{\Theta_2^*(0,m)}b_2 g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2\right) \\ +q'\left(\int_{\Theta_2^*(0,m)}^{+\infty}\ell_2 g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)d\Theta_2-\int_{-\infty}^{\Theta_2^*(0,m)}b_2 g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)d\Theta_2\right) \\ +(1-p'-q')\left(\int_{\Theta_2^*(0,m)}^{+\infty}\ell_2 g(x_U^*|\Theta_2)f(\Theta_2|0)d\Theta_2-\int_{-\infty}^{\Theta_2^*(0,m)}b_2 g(x_U^*|\Theta_2)f(\Theta_2|0)d\Theta_2\right) \end{pmatrix}
\end{aligned}$$

where we used that $\frac{x_I^*(0,m)}{dp}=0,\forall m$. Moreover:

$$\frac{dp'}{dp} \;=\; \frac{\phi\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu+\Delta))\right)\begin{pmatrix} \phi\left(\sqrt{\alpha_1}(\Theta_1^*-\mu)\right) \\ -\sqrt{\alpha_1}\frac{d\Theta_1^*}{dp}\frac{q\phi\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu-s\Delta))\right)+(1-p-q)\phi\left(\sqrt{\alpha_1}(\Theta_1^*-\mu)\right)}{1/\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu+\Delta))\right)} \\ -\sqrt{\alpha_1}\frac{d\Theta_1^*}{dp}\left(\frac{q\phi\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu-s\Delta))\right)}{1/\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu-s\Delta))\right)}+\frac{(1-p-q)\phi\left(\sqrt{\alpha_1}(\Theta_1^*-\mu)\right)}{1/\left(\sqrt{\alpha_1}(\Theta_1^*-\mu)\right)}\right) \end{pmatrix}}{\left(p\phi\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu+\Delta))\right)+q\phi\left(\sqrt{\alpha_1}(\Theta_1^*-(\mu-s\Delta))\right)+(1-p-q)\phi\left(\sqrt{\alpha_1}(\Theta_1^*-\mu)\right)\right)^2}$$

For the limit $\alpha_1\to 0$ we have that $\frac{d\Theta_1^*}{dp}\to 0$, $p'\to p$ and $q'\to q$ so that $\frac{dp'}{dp}\to 1$ and $\frac{dq'}{dp}\to\frac{1}{s}$.

Next, we analyze $\frac{d\Theta_2^*(0,m)}{dp}$ which also gives us $\frac{dx_U^*(0)}{dp}$. Both derivatives are arbitrarily close to zero for sufficiently high s and μ. This means that we can focus on the first summands of $dv(0,1)/dp$ associated with dp'/dp. Taking the derivative with respect to Δ we can see that $dv(0,1)/dp > 0$ for sufficiently small values of Δ. Taken together the result of Corollary 2 holds under Assumption 1 and sufficiently small values of α_1 and Δ. This concludes the proof.

A.7 Proof of Proposition 4

We define the differential ex-ante probability of regime change, D, between when investors acquire information after observing a crisis in region 1, $n_2 = 1$, and when investors do not, $n_2 = 0$ (e.g., because of a high c). Recall that $\Theta_2^*(0,m) = \Theta_U^*$, $\forall m \in \{\Delta, -s\Delta, 0\}$, which solves equation (21). Importantly, investors do not acquire information in both scenarios after not observing a crisis in region 1, $f = 0$, which helps to simplify the expression. Suppose that $\mu = \gamma = p = \frac{1}{2}$ and $s = 1$, which implies $1 - p - q = 0$. Then D can be written as:

$$\begin{aligned} D \equiv & \left(\Pr\{m=\Delta|f=1\}\Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} + \Pr\{m=-s\Delta|f=1\}\Pr\{\Theta_2 < \Theta_2^*(1,-s\Delta)\}\right) \\ & - \left(\Pr\{m=\Delta|f=1\}\Pr\{\Theta_2 < \Theta_2^*(0,\Delta)|f=1\} + \Pr\{m=-s\Delta|f=1\}\Pr\{\Theta_2 < \Theta_2^*(0,-s\Delta)|f=1\}\right) \end{aligned}$$

We divide by $\Pr\{\Theta_1 < \Theta_1^*|m=\Delta\} + \Pr\{\Theta_1 < \Theta_1^*|m=-s\Delta\}/s$, then $D > 0$ if and only if:

$$D' \equiv \frac{\left(-\Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} + \Pr\{\Theta_2 < \Theta_2^*(0,\Delta)|f=1\}\right)\Pr\{\Theta_1 < \Theta_1^*|m=\Delta\}}{\left(\Pr\{\Theta_2 < \Theta_2^*(1,-\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,-\Delta)|f=1\}\right)\Pr\{\Theta_1 < \Theta_1^*|m=-\Delta\}} < 1.$$

Whether or not $D' < 1$ is determined by the relative weighting of the differential crisis probabilities in region 2 by the conditional crisis probabilities in region 1. Note that $D' = 1$ and $D = 0$ if $\alpha_2 = 0$ so that the prior of Θ_2 is improper uniformly distributed. We next argue that $D' < 1$ for small, but positive, values of $\alpha_2 = 0$ where the weighting by the conditional crisis probabilities in region 1 dominates with $\Pr\{\Theta_1 < \Theta_1^*|m=\Delta\} < \Pr\{\Theta_1 < \Theta_1^*|m=-s\Delta\}$.

Using L'Hôpital's rule we can derive:

$$\lim_{\alpha_2 \to 0} D' = \lim_{\alpha_2 \to 0} \frac{\Phi(-\Delta\sqrt{\alpha_1})}{\Phi(\Delta\sqrt{\alpha_1})} \frac{\frac{1}{2}\left(\Theta_2^*(0,\Delta) - \Theta_2^*(1,\Delta)\right) - \alpha_2\left(\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} - \frac{d\Theta_2^*(0,\Delta)}{d\alpha_2}\right)}{\frac{1}{2}\left(\Theta_2^*(1,-\Delta) - \Theta_2^*(0,-\Delta)\right) + \alpha_2\left(\frac{d\Theta_2^*(1,-\Delta)}{d\alpha_2} - \frac{d\Theta_2^*(0,-\Delta)}{d\alpha_2}\right)}.$$

Note that $\lim_{\alpha_2 \to 0} \Theta_2^*(0,m) = \lim_{\alpha_2 \to 0} \Theta_2^*(1,m) = \frac{1}{2}$ and $\lim_{\alpha_2 \to 0}\left(\frac{d\Theta_2^*(1,m)}{d\alpha_2} - \frac{d\Theta_2^*(0,m)}{d\alpha_2}\right)$ is finite so that both, the nominator and denominator go to zero in the limit. Hence, we apply L'Hôpital's rule another time and plug in for the derivatives of the fundamental thresholds to arrive at:

$$\lim_{\alpha_2 \to 0} D' = \lim_{\alpha_2 \to 0} \frac{\Phi(-\Delta\sqrt{\alpha_1})}{\Phi(\Delta\sqrt{\alpha_1})} \frac{-\left(\frac{3}{2}+\alpha_2\right)\frac{\Theta^*(1,\Delta)-\left(\frac{1}{2}+\Delta\right)}{\alpha_2-\sqrt{\beta}/\phi(\Phi^{-1}(\Theta_2^*(1,\Delta)))} - \left(\frac{3}{2}+\alpha_2\right)\frac{dJ_2/d\alpha_2}{dJ_2/d\Theta_2^*(0,\Delta)}}{\left(\frac{3}{2}+\alpha_2\right)\frac{\Theta^*(1,-\Delta)-\left(\frac{1}{2}-\Delta\right)}{\alpha_2-\sqrt{\beta}/\phi(\Phi^{-1}(\Theta_2^*(1,-\Delta)))} + \left(\frac{3}{2}+\alpha_2\right)\frac{dJ_2/d\alpha_2}{dJ_2/d\Theta_2^*(0,-\Delta)}} < 1.$$

First, $\Phi(-\Delta\sqrt{\alpha_1}) < \Phi(\Delta\sqrt{\alpha_1})$. In the limit the nominator and denominator of the second factor take on an identical finite non-zero value. Taken together, $\lim_{\alpha_2 \to 0} D' < 1$ and $\lim_{\alpha_2 \to 0} D > 0$. As a result, we can show by continuity that there exists a $\overline{\alpha_2} > 0$ such that, for all $\alpha_2 < \overline{\alpha_2}$, the ex-ante probability of regime change is higher for $c \in (\underline{c}, \overline{c})$ than for $c > \overline{c}$ when investors don't acquire information (in the special case of $\mu = \gamma = p = \frac{1}{2}$ and $s = 1$). The result in Proposition 4 follows.

Using a similar argument, it can be shown that the opposite result can arise. This is the case for the limit when $\alpha_1 \to \infty$. Intuitively, this time the relative weighting of the differential crisis probabilities in region 2 dominate, with $(-\Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} + \Pr\{\Theta_2 < \Theta_2^*(0,\Delta)|f = 1\}) > (\Pr\{\Theta_2 < \Theta_2^*(1,-\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,-\Delta)|f = 1\})$ if $\alpha_1 > 0$ so that $\lim_{\alpha_1 \to \infty} D' > 1$.

A.8 Proof of Proposition 5

We analyze the role of transparency for the case of $\mu = \gamma = \frac{1}{2}$ and $1 - p - q = 0$. First, we establish some results that will be useful. It can be shown that $\frac{df(\Theta_2^*|m)}{d\alpha_2} < 0, \forall m \in \{\Delta, 0, -s\Delta\}$. Moreover, $\frac{d\Theta_2^*(m)}{d\alpha_2} < 0$ and $\frac{dx_I^*(m)}{d\alpha_2} < 0$ for $m = \Delta$, as well as $\frac{d\Theta_2^*(m)}{d\alpha_2} > 0$ and $\frac{dx_I^*(m)}{d\alpha_2} > 0$ for $m = -s\Delta$.

Next, we analyze the derivative of the value of information with respect to α_2. Observe that p' and q' only depend on α_1 and not on α_2. For $1 - p - q = 0$ we can focus on the derivatives of the first and second summand of equation (40) to describe the incentives to become informed:

$$
\begin{aligned}
\frac{dv(1,f)}{d\alpha_2} &= p'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) b_2 d\Theta_2 \\ +\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_I^*(1,\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|\Delta) b_2 d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) \ell_2 d\Theta_2 \\ -\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_I^*(1,\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|\Delta) \ell_2 d\Theta_2 \end{pmatrix} \\
&+ p'(f)\begin{pmatrix} -\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|\Delta) b_2 \\ -\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|\Delta) \ell_2 \end{pmatrix} \\
&+ p'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|\Delta)}{d\alpha_2} b_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|\Delta)}{d\alpha_2} \ell_2 \end{pmatrix} \\
&+ q'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|-s\Delta) b_2 d\Theta_2 \\ +\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_I^*(1,-s\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|-s\Delta) b_2 d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|-s\Delta) \ell_2 d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_I^*(1,-s\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|-s\Delta) \ell_2 d\Theta_2 \end{pmatrix} \\
&+ q'(f)\begin{pmatrix} -\frac{d\Theta_2^*(1,-s\Delta)}{d\alpha_2} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|-s\Delta) b_2 \\ -\frac{d\Theta_2^*(1,-s\Delta)}{d\alpha_2} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|-s\Delta) \ell_2 \end{pmatrix} \\
&+ q'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|-s\Delta)}{d\alpha_2} b_2 d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|-s\Delta)}{d\alpha_2} \ell_2 d\Theta_2 \end{pmatrix}.
\end{aligned}
$$

For the special case with $\mu = \gamma = \frac{1}{2}$ and $s = 1$ the derivative simplifies, because $\frac{x_I^*(1,\Delta)}{d\alpha_2} = -\frac{x_I^*(1,-\Delta)}{d\alpha_2}$,

$\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} = -\frac{d\Theta_2^*(1,-\Delta)}{d\alpha_2}$, $\frac{1}{2} - \Theta_2^*(1,\Delta) = \Theta_2^*(1,-\Delta) - \frac{1}{2}$:

$$
\begin{aligned}
\frac{dv(1,f)}{d\alpha_2}\Big|_{\mu=\gamma=\frac{1}{2},s=1} \propto\ & \frac{dx_I^*(1,\Delta)}{d\alpha_2}\left(\int_{-\infty}^{\Theta_2^*(1,\Delta)} g(x_I^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 - \int_{\Theta_2^*(1,\Delta)}^{+\infty} g(x_I^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \right) \\
& - p'(f)\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2}\begin{pmatrix} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2|\Delta) \\ +\int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|\Delta) \end{pmatrix} \\
& - q'(f)\frac{d\Theta_2^*(1,-\Delta)}{d\alpha_2}\begin{pmatrix} \int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|-\Delta) \\ +\int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|-\Delta) \end{pmatrix} \\
& + p'(f)\frac{dx_U^*(1,f)}{d\alpha_2}\left(\int_{\Theta_2^*(1,\Delta)}^{+\infty} g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 - \int_{-\infty}^{\Theta_2^*(1,\Delta)} g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \right) \\
& + q'(f)\frac{dx_U^*(1,f)}{d\alpha_2}\begin{pmatrix} \int_{\Theta_2^*(1,-\Delta)}^{+\infty} g(x_U^*|\Theta_2)f(\Theta_2|-\Delta)d\Theta_2 \\ -\int_{-\infty}^{\Theta_2^*(1,-\Delta)} g(x_U^*|\Theta_2)f(\Theta_2|-\Delta)d\Theta_2 \end{pmatrix} \\
& + p'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)}\int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2)dx_{i2}\frac{-\alpha_2 y(\Theta_2)}{2}f(\Theta_2|\Delta)d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty}\int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2)dx_{i2}\frac{-\alpha_2 y(\Theta_2)}{2}f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix} \\
& + q'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-\Delta)}\int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2)dx_{i2}\frac{-\alpha_2 y(\Theta_2)}{2}f(\Theta_2|-\Delta)d\Theta_2 \\ +\int_{\Theta_2^*(1,-\Delta)}^{+\infty}\int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2)dx_{i2}\frac{-\alpha_2 y(\Theta_2)}{2}f(\Theta_2|-\Delta)d\Theta_2 \end{pmatrix},
\end{aligned}
$$

where $y(\Theta_2) \equiv \Theta_2 - \frac{1}{2} + \Delta$. We can show that the first summand is zero. To see this, we rewrite the integrand and evaluate it at $x^*(1,\Delta)$:

$$
\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{\sqrt{\alpha_2\beta}}{2\pi}e^{-\frac{1}{2}u(\Theta_2)}d\Theta_2 - \int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{\sqrt{\alpha_2\beta}}{2\pi}e^{-\frac{1}{2}u(\Theta_2)}d\Theta_2, \tag{46}
$$

where $u(\Theta_2) \equiv [\beta(\Theta_2^*(1,\Delta) + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Theta_2^*(1,\Delta)) - \Theta_2)^2 + \alpha_2(\Theta_2 - \frac{1}{2} - \Delta)^2]$. From the equilibrium condition we have $\frac{1}{\sqrt{\beta}}\Phi^{-1}(\Theta_2^*(1,\Delta)) = \frac{\alpha_2}{\beta}y(\Theta_2^*(1,\Delta))$. Moreover, matching fundamental realizations that are equidistant from the fundamental equilibrium threshold we find that:

$$
\begin{aligned}
\frac{d\left(e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} - e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)}\right)}{d\varepsilon} &= -\frac{1}{2}\frac{du(\Theta_2^*-\varepsilon)}{d\varepsilon}e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} + \frac{1}{2}\frac{du(\Theta_2^*+\varepsilon)}{d\varepsilon}e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)} \\
&= -\frac{1}{2}[2\beta(\frac{\alpha_2}{\beta}y(\Theta_2^*)+\varepsilon) - 2\alpha_2(y(\Theta_2^*)-\varepsilon)]e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} \\
&\quad +\frac{1}{2}[-2\beta(\frac{\alpha_2}{\beta}y(\Theta_2^*)-\varepsilon) + 2\alpha_2(y(\Theta_2^*)+\varepsilon)]e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)} \\
&= (\alpha_2+\beta)\varepsilon\left(-e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} + e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)}\right) = 0, \forall\varepsilon \geq 0.
\end{aligned}
$$

Next, the second and third summands are strictly positive since $x_I^*(1,-s\Delta) > x_U^*(1,f) > x_I^*(1,\Delta)$.

For the remaining summands we consider the limit case $\alpha_1 \to 0$, which vastly simplifies the analysis. In the limit case region 1 becomes irrelevant and $p'(f) = q'(f) = p$. Summands four and five are zero since $x_U^*(1,f) = \frac{1}{2}, \forall f \in \{0,1\}$. Finally, summands six and seven have an ambiguous sign and we know that $df(\Theta_2|\Delta)/d\alpha_2 = df(\Theta_2|-\Delta)/d\alpha_2 < 0$ for $\mu = \gamma = \frac{1}{2}$ and $s = 1$. For the limit $\alpha_1 \to 0$ we can rewrite summands six and seven as:

$$\frac{\alpha_2}{2}\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(1,\Delta)}\left(\int_{x_I^*(1,\Delta)}^{\Theta_2^*(1,\Delta)} g(x_{i2}|\Theta_2)dx_{i2}d\Theta_2 + \int_{\Theta_2^*(1,\Delta)}^{1/2} g(x_{i2}|\Theta_2)dx_{i2}\right) y(\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(1,\Delta)}^{+\infty}\left(\int_{x_I^*(1,\Delta)}^{\Theta_2^*(1,\Delta)} g(x_{i2}|\Theta_2)dx_{i2} + \int_{\Theta_2^*(1,\Delta)}^{1/2} g(x_{i2}|\Theta_2)dx_{i2}\right) y(\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \end{array}\right) \tag{47}$$

$$= \frac{\alpha_2^{3/2}\beta^{1/2}}{4\pi}\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(1,\Delta)}\left(\begin{array}{c} \int_{\Theta_2^*(1,\Delta)}^{1/2} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2} \\ +\int_{\Theta_2^*(1,\Delta)+\frac{\alpha_2}{\beta}\left(\Theta_2^*(1,\Delta)-\frac{1}{2}-\Delta\right)}^{\Theta_2^*(1,\Delta)} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2} \end{array}\right) y(\Theta_2)d\Theta_2 \\ -\int_{\Theta_2^*(1,\Delta)}^{+\infty}\left(\begin{array}{c} \int_{\Theta_2^*(1,\Delta)}^{1/2} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2} \\ +\int_{\Theta_2^*(1,\Delta)+\frac{\alpha_2}{\beta}\left(\Theta_2^*(1,\Delta)-\frac{1}{2}-\Delta\right)}^{\Theta_2^*(1,\Delta)} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2} \end{array}\right) y(\Theta_2)d\Theta_2 \end{array}\right).$$

We first derive sufficient conditions such that a combination of the first and third summands in the last expression of equation (47) is positive. We again match equidistant fundamental realizations and evaluate at the signal $x_{i2} = \Theta_2^*(1,\Delta)$. If the resulting expression is positive at $x_{i2} = \Theta_2^*(1,\Delta)$, then also the combination of the first and third summands must be positive:

$$I(\varepsilon) = I_1(\varepsilon) + I_2(\varepsilon) \equiv (\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\varepsilon^2+\alpha_2(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2]} - (\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\varepsilon^2+\alpha_2(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2]}.$$

We find that $\int_0^{+\infty} I(\varepsilon)d\varepsilon > 0$ for sufficiently high values of Δ. First, $I_1(0) = I_2(0) = 0$. Taking derivatives leads to:

$$\begin{aligned} \frac{d\left(\frac{I_1(\varepsilon)}{e^{-\frac{1}{2}\beta\varepsilon^2}}\right)}{d\varepsilon} &= (1 - \alpha_2(\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta)^2)e^{-\frac{\alpha_2}{2}(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2} \\ \frac{d\left(\frac{I_2(\varepsilon)}{e^{-\frac{1}{2}\beta\varepsilon^2}}\right)}{d\varepsilon} &= (\alpha_2(\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta)^2 - 1)e^{-\frac{\alpha_2}{2}(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2}, \end{aligned}$$

For sufficiently high Δ we have that $dI_1(\varepsilon)/d\varepsilon < 0$, while $dI_2(\varepsilon)/d\varepsilon > 0$ for small and high ε. Only for the intermedate range $\varepsilon \in \left(\hat{\varepsilon} - \sqrt{1/\alpha_2}, \hat{\varepsilon} + \sqrt{1/\alpha_2}\right)$ we have that $I_2(\varepsilon) < 0$, where $\hat{\varepsilon} \equiv$

$\frac{1}{2}+\Delta-\Theta_2^*(1,\Delta)$. Note that $\int_0^{\hat{\varepsilon}-\sqrt{1/\alpha_2}} I(\varepsilon)d\varepsilon > 0$ and $\int_{\hat{\varepsilon}+\sqrt{1/\alpha_2}}^{+\infty} I(\varepsilon)d\varepsilon > 0$ if Δ is high, while $\int_{\hat{\varepsilon}-\sqrt{1/\alpha_2}}^{\hat{\varepsilon}+\sqrt{1/\alpha_2}} I(\varepsilon)d\varepsilon < 0$. It can be shown that $\int_0^{+\infty} I_2(\varepsilon)d\varepsilon > 0$ and $\int_0^{+\infty} I_2(\varepsilon)d\varepsilon > -\int_0^{+\infty} I_1(\varepsilon)d\varepsilon$ for sufficiently high Δ and small α_2. To see this, observe that:

$$-\int_0^{\sqrt{1/\alpha_2}} e^{-\frac{\alpha_2}{2}\varepsilon'^2}d\varepsilon' + \int_{\sqrt{1/\alpha_2}}^{+\infty} e^{-\frac{\alpha_2}{2}\varepsilon'^2}d\varepsilon' > 0 \Leftarrow \alpha_2 < \left(\sqrt{\frac{\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right)\right)^{-2} \approx 0.86. \quad (48)$$

Finally, we consider the combination of the second and fourth summands in the last expression of equation (47). Following a similar arugment as before, we can show that it is positive for sufficiently high Δ. To see this, we again match equidistant fundamental realizations and evaluate at the signal $x_{i2} = \Theta_2^*(1,\Delta) + \frac{\alpha_2}{\beta}\left(\Theta_2^*(1,\Delta) - \frac{1}{2} - \Delta\right)$:

$$\begin{aligned}
I(\varepsilon) = I_3(\varepsilon) + I_4(\varepsilon) &\equiv (\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)+\varepsilon\right)^2+\alpha_2(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2]} \\
&- (\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)-\varepsilon\right)^2+\alpha_2(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2]} \\
\frac{dI_3(\varepsilon)}{d\varepsilon} &= -(1+(\alpha_2+\beta)\varepsilon(\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta))e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)+\varepsilon\right)^2+\alpha_2(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2]} \\
\frac{dI_4(\varepsilon)}{d\varepsilon} &= -(1-(\alpha_2+\beta)\varepsilon(\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta))e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)-\varepsilon\right)^2+\alpha_2(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2]}.
\end{aligned}$$

We have that $I(0) = 0$ and $I_3(\varepsilon) < 0, \forall\varepsilon > 0$. Moreover, $I_3(\varepsilon) > 0$ for small values of ε and $I_3(\varepsilon) < 0$ for large values of ε. Similar to before, we can show that also $\int_0^{+\infty}(I_3+I_4)d\varepsilon > 0$ for sufficiently high Δ and small α_2.

Taken together, $\frac{dv(1,f)}{d\alpha_2} > 0$ for the special case with $\mu = \gamma = \frac{1}{2}$, $s = 1$, $1-p-q = 0$ and sufficiently high Δ and sufficiently small α_1 and α_2.

A.9 Figure 2: Comparative statics

We examine how the differential value of information changes in the parameter s, $\frac{d[v(1,1)-v(1,0)]}{ds}$. We consider the special case of Figures 1-3, where $\mu = \gamma = \frac{1}{2}$. To further simplify the analysis, we set $s = 1$ and $1 - p - q = 0$. It follows that:

$$\begin{aligned}
\frac{d[v(1,1)-v(1,0)]}{ds} &= p'(1)\left(\begin{array}{l} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \end{array}\right) \\
&+ q'(1)\left(\begin{array}{l} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{array}\right) \\
&- p'(0)\left(\begin{array}{l} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \end{array}\right) \\
&- q'(0)\left(\begin{array}{l} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{array}\right) \\
&+ q'(1)\left(\begin{array}{l} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{array}\right) \\
&- q'(0)\left(\begin{array}{l} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{array}\right) \\
&+ q'(1)\left(\frac{d\Theta_2^*(1,-s\Delta)}{ds}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|-s\Delta) \right) \\
&- q'(0)\left(\frac{d\Theta_2^*(1,-s\Delta)}{ds}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|-s\Delta) \right) \\
&+ q'(1)\left(\begin{array}{l} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \end{array}\right) \\
&- q'(0)\left(\begin{array}{l} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \end{array}\right), \qquad (49)
\end{aligned}$$

where we used that, for $\mu = \frac{1}{2}$ and $s = 1$, we have $\frac{1}{2} - \Theta_2^*(1,\Delta) = \Theta_2^*(1,-s\Delta) - \frac{1}{2}$ and $\frac{1}{2} - x_I^*(1,\Delta) = x_I^*(1,-s\Delta) - \frac{1}{2}$. Moreover, $\Theta_2^*(1,0) = \Theta_1 = \frac{1}{2}$, $\frac{dp'(f=0)}{ds} = \frac{dq'(f=1)}{ds}$, $\frac{dp'(f=1)}{ds} = \frac{dq'(f=0)}{ds}$ and $x_U^*(n = 1, f = 1) - \frac{1}{2} = \frac{1}{2} - x_U^*(n = 1, f = 0)$ due to symmetry in the special case. Finally, together with $1 - p - q = 0$, we have that $\frac{dp'(f)}{ds} = -\frac{dq'(f)}{ds}, \forall f \in \{0,1\}$.

Note that summands 5-6 of equation (49) are jointly positive since the expressions inside the brackets are positive, $q'(1) > q'(0)$ and $\frac{dx_I^*(1,-s\Delta)}{ds} > 0$. Given that $\frac{d\Theta_1^*(1,-s\Delta)}{ds} > 0$, summands 7-8 are jointly positive if:

$$\frac{q'(1)}{q(0)} > \frac{\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}}{\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}},$$

which holds for sufficiently high β. To see this, observe that a higher β shifts more mass to the upper signal threshold because $x_I^*(1,-s\Delta)$ and $\Theta_2^*(1,-s\Delta)$ approach each other faster than $\Theta_2^*(1,-s\Delta)$ and $x_U^*(1,f)$ if β increases. Formally, $x_I^*(1,-s\Delta)-\Theta_2^*(1,-s\Delta) = \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\Theta_2^*(1,-s\Delta)\right) < \Theta_2^*(1,-s\Delta)-\Theta_2^*(0,f)-\frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\Theta_2^*(0,f)\right)$ for sufficiently high β.

Similarly, summands 9-10 are jointly positive if:

$$\begin{aligned} & q'(1)\left(\begin{array}{l} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \end{array}\right) \\ - \quad & q'(0)\left(\begin{array}{l} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \end{array}\right) > 0. \end{aligned}$$

Notably, the second summands in both brackets are positive and the first summands are negative for values $\Theta_2 \in \left(\frac{1}{2}-s\Delta, \Theta_2^*(1,-s\Delta)\right)$ and positive for $\Theta_2 < \frac{1}{2}-s\Delta$. For a given Δ we have that the probability weight in the positive regions dominates if α sufficiently small, which makes the expressions in the brackets positive. Given that, we can use an analog argument to show that the inequality holds for sufficiently high β.

Finally, we derive conditions such that also summands 1-4 of equation (49) are jointly positive. Observe that for large β the second summands of the first bracket and the first summand of the fourth bracket dominate. Moreover, for large Δ the second summand of the second bracket, as well as the first summand of the third bracket dominate. In addition, a large Δ implies that $x_U^*(1,1) \to x_I^*(1,-s\Delta)$ and $x_U^*(1,0) \to x_I^*(1,\Delta)$ so that $\frac{dx_U^*(1,1)}{ds} > 0$ and $\frac{dx_U^*(1,0)}{ds} \to 0$. Taken together, summands 1-4 are jointly positive for sufficiently high β and Δ.

In conclusion we have shown that $\frac{d[v(1,1)-v(1,0)]}{ds} > 0\big|_{s=1}$ for the special case $\mu=\gamma=p=\frac{1}{2}$ as in Figure 2 and under the sufficient condition that β and Δ are high.

A.10 Skewness of the macro shock

This section elaborates on the discussion in Section 5.2. We first consider our baseline model and revisit the key results of this paper (differential information choice and wake-up call contagion effect) for the special case of $\mu = \gamma = \frac{1}{2}$. As demonstrated in Figure 2 and in Proposition 2, the differential information choice arises for sufficiently high values of s, giving rise to the wake-up call contagion effect in Proposition 3. We first show that the differential information choice hinges on $s > 1$ by considering the case when $s = 1$. Observe that for $s = 1$ the result of Lemma 4 continues to be valid, which implies that the first and third inequality of (7) in Proposition 2 hold. However, the second inequality of (7) is violated as we show in the proof of Corollary 3.

A.10.1 Proof of Corollary 3

Consider the equilibrium condition for region 1 in equation (13) and observe that its structure is fully symmetric with $\Theta_1^* = x_1^* = \frac{1}{2}$ when $\mu = \gamma = \frac{1}{2}$ and $s = 1$. As a result, the updated prior beliefs about the macro shock distribution p' and q' are exact mirror images when observing a crisis in region 1 or not. Based on this results, the structure of the equilibrium condition for region 2 in equation (21) also shows exact mirror images when comparing the two scenarios. The same type of symmetry statement can be made for the signal thresholds. Applying these results to the value of information in equation (40), we find that there is no differential value of information: $v(n_2, 0) = v(n_2, 1)$, $\forall n_2 \in [0, 1]$. This concludes the proof of Corollary 3.

A.10.2 Offsetting changes in Δ

Following an analogous argument as in Corollary 3 we can use the symmetry properties to show that there is no differential information choice for $\mu = \gamma = \frac{1}{2}$ and $s = 1$. For the general case with $s > 1$, we revisit Proposition 2. Observe that the result of Lemma 4 is unaffected by the modification of the model. However, some steps in the proof of Proposition 2 need to be adjusted:

Step 1: The first term in brackets is now directly affected by s through $x_U^*(1)$ and indirectly through $x_I^*(1, \Delta(s))$ and $\Theta_I^*(1, \Delta(s))$. Second, observe that $\Theta_I^*(1, \Delta(s))$ is growing strictly larger in s since $\frac{ds\Delta(s)}{ds} > 0$. As before, $x_U^*(1) \to x_I^*(1, \Delta)$ as $s \to \infty$. Hence, $v(1, f = 0) \to 0$.

Steps 2, 3 and 4: After small adjustments the results in the proof of Proposition 2 go through.

Hence, the third inequality of (7) in Proposition 2 follows.

To conclude, the two key insights on the differential information choice and the wake-up call contagion effect remain valid for the modified model with offsetting changes in Δ under sufficient conditions akin to Assumption 1.

A.10.3 Independent s and q

We first discuss the information choice and, thereafter, the wake-up call contagion effect.

When s and q vary independently, we have $E[m] = 0$ for $s = 1$ and $E[m] < 0$ for $s > 1$. In the former case the model is unchanged and the analysis of Corollary 3 applies. For $s > 1$, we revisit inequality (7) in Proposition 2. The result of Lemma 4 is again unaffected by the modification of the model, while the proof of Proposition 2 needs some adjustments:

Step 1: Despite q not being anymore affected by s, q' is still affected via $\Pr\{f, m = -s\Delta\}$. As before, $x_U^*(1) \to x_I^*(1,\Delta)$ as $s \to \infty$ and $v(1, f = 0) > v(0, f = 0) \to 0$.

Step 2: For $f = 1$ we now have that:

$$\frac{\partial}{\partial s}\left(\frac{q'}{p'} = \frac{q}{p}\frac{\Pr\{f = 1, m = -s\Delta\}}{\Pr\{f = 1|m = \Delta\}}\right) > 0.$$

The result flips if $f = 0$.

Step 3 and 4: Observe that, for a given μ and $s > 1$, the event of a negative macro shock is never considered to be the most probable state of the world, i.e. $q' < p'$, provided that q is sufficiently low: $\frac{p}{q} \geq \Pr\{f|m = -s\Delta\}(\Pr\{f|m = \Delta\})^{-1}$. Moreover $[q'|f = 1] >> 0$, while $[q'|f = 0]$ is arbitrarily small for high values of μ. Again, both the first and second summand of $v(0, f = 1)$ must be strictly positive and away from zero, since $x_U^* \nrightarrow x_I^*(0,\Delta)$ and $x_U^* \nrightarrow x_I^*(0,-s\Delta)$. By continuity, the result also holds for large, but finite, values of μ and s, as well as for sufficiently small $1 - p - q$. Hence, $v(n_2 = 0, f = 1) > v(n_2 = 1, f = 0)$ and inequality (7) follows.

In sum, the results on the information choice go through for sufficient conditions akin to Assumption 1 with the addition that q is sufficiently low.

We next turn to the wake-up call contagion result in Proposition 3. We face the challenge that the biased macro shock can lead to an opposing effect. Nevertheless, using the same parameters as in Figure 1, the left panel of Figure 5 illustrates that the total effect has the desired sign such that inequality (8) of Proposition 3 continues to hold for all $s > 1$: we have that $\Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\} > \Pr\{\Theta_2 < \Theta_U^*|f = 0\}$. If Δ is higher, the right-hand side of the inequality is lower for all values of s, because a higher positive macro shock leads to a more favorable belief about Θ_2 after not observing a crisis in region 1.

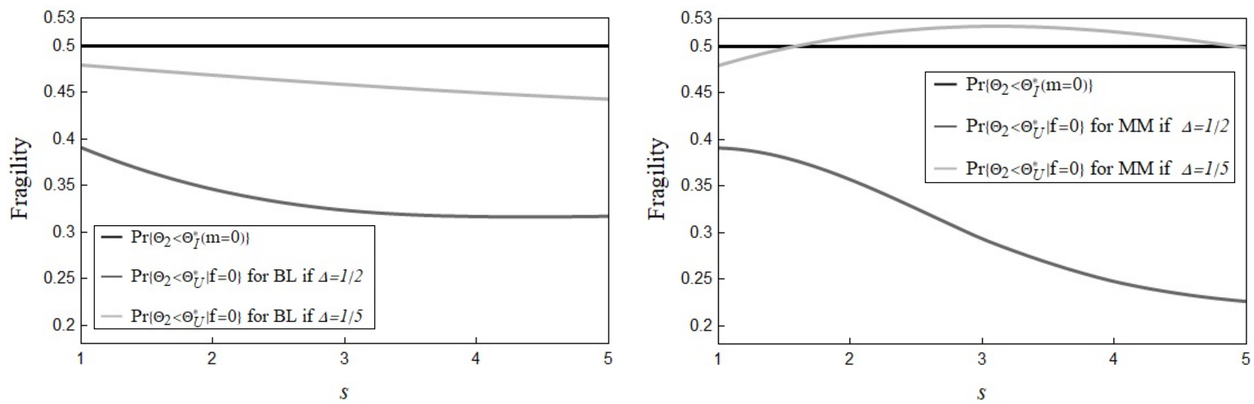

Figure 5: Wake-up call contagion for the baseline (BL) and for a modified model (MM) where s and q can vary independently. We vary s from 1 to 5 and compare the crisis probabilities in region 2. As in Figure 3, $\Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\} = 1/2$ is unaffected and identical in both models [black solid line]. In the left (right) panel we draw $\Pr\{\Theta_2 < \Theta_U^*|f = 0\}$ for the baseline (modified) model if $\Delta = 1/2$ [dark grey] and if $\Delta = 1/5$ [light grey]. For $s = 1$ the probability of regime change is identical. The other parameter values are as in Figure 1.

As a result, the wake-up call contagion effect can prevail in the modified model. This result is, however, not guaranteed to hold. When reducing Δ from $1/2$ to $1/5$ in the right panel of Figure 5 we can see that inequality (8) is for some intermediate values of s violated in the modified model where s and q can vary independently (light grey line).